

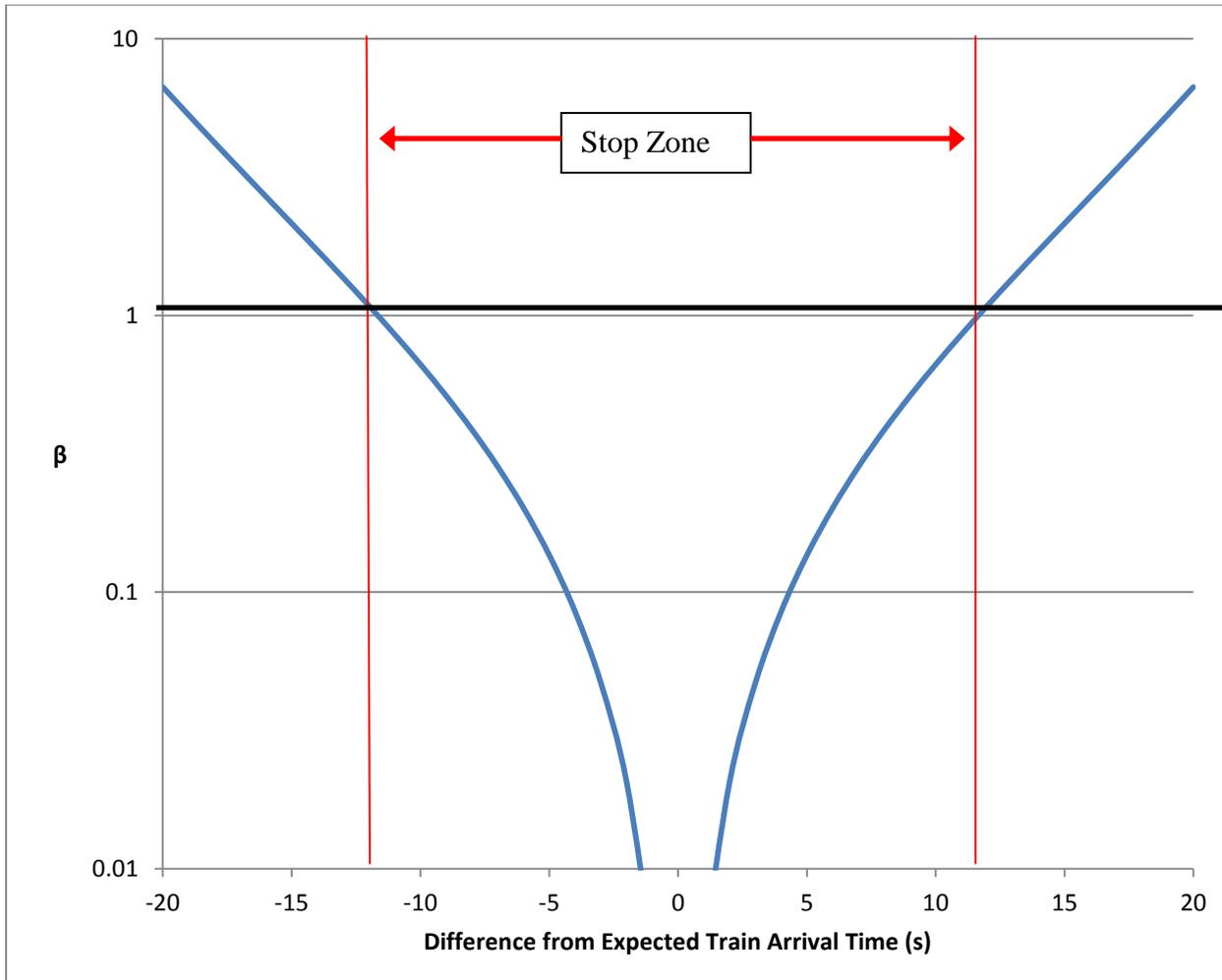


U.S. Department of  
Transportation

Federal Railroad  
Administration

# Using Signal Detection Theory to Understand Grade Crossing Warning Time and Motorist Stopping Behavior

Office of Research  
and Development  
Washington, DC 20590



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13. ABSTRACT (Maximum 200 words) Motorist error or poor judgment is a significant causal factor in highway-rail grade crossing collisions. Crashes at grade crossings equipped with warning devices often involve motorists who drive around gates or across railroad tracks while flashing lights are warning them that a train is approaching. This noncompliant behavior may be due to the motorists' expectations of train arrival time following activation of gates and lights as well as the overall duration of the warning. Because warning times are variable, it is uncertain whether the mean warning duration, the variability of the warning's duration, or both are influencing motorists' decisions to disregard the warnings. As a result, signal detection theory was used to model motorists' stopping behavior at active grade crossings. The key factor in predicting motorist stopping behavior is treating the subjective probability that a train is in the grade crossing as a function of the expected arrival time of the train and this was modeled with Gaussian, Chi-squared and Poisson probability distributions. The probability of stopping predicted from each probability distribution was compared with data collected by Richards and Heathington (1990). The Gaussian model provided the best fit to the data and indicated that warning time variability is the most important factor affecting motorist stopping behavior. Additional data collection to confirm and refine the model is discussed. A theory of motorist behavior at grade crossings, such as signal detection theory, provides a means to critically examine inchoate hypotheses so that they can be more formally stated and vigorously tested. This theory should continue to be developed for evaluating motorist behavior at grade crossings.					
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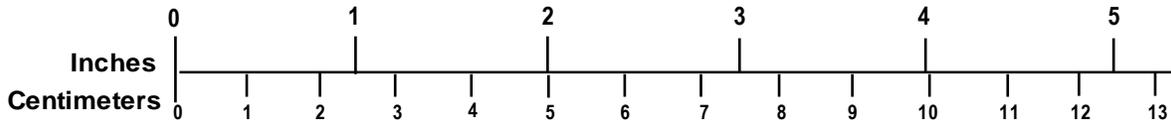
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## ENGLISH TO METRIC

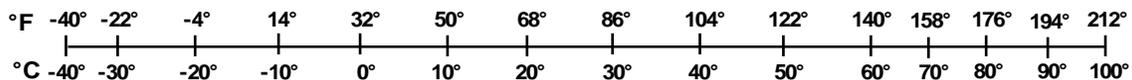
## METRIC TO ENGLISH

<p><b>LENGTH (APPROXIMATE)</b></p> <p>1 inch (in) = 2.5 centimeters (cm)</p> <p>1 foot (ft) = 30 centimeters (cm)</p> <p>1 yard (yd) = 0.9 meter (m)</p> <p>1 mile (mi) = 1.6 kilometers (km)</p>	<p><b>LENGTH (APPROXIMATE)</b></p> <p>1 millimeter (mm) = 0.04 inch (in)</p> <p>1 centimeter (cm) = 0.4 inch (in)</p> <p>1 meter (m) = 3.3 feet (ft)</p> <p>1 meter (m) = 1.1 yards (yd)</p> <p>1 kilometer (km) = 0.6 mile (mi)</p>
<p><b>AREA (APPROXIMATE)</b></p> <p>1 square inch (sq in, in<sup>2</sup>) = 6.5 square centimeters (cm<sup>2</sup>)</p> <p>1 square foot (sq ft, ft<sup>2</sup>) = 0.09 square meter (m<sup>2</sup>)</p> <p>1 square yard (sq yd, yd<sup>2</sup>) = 0.8 square meter (m<sup>2</sup>)</p> <p>1 square mile (sq mi, mi<sup>2</sup>) = 2.6 square kilometers (km<sup>2</sup>)</p> <p>1 acre = 0.4 hectare (he) = 4,000 square meters (m<sup>2</sup>)</p>	<p><b>AREA (APPROXIMATE)</b></p> <p>1 square centimeter (cm<sup>2</sup>) = 0.16 square inch (sq in, in<sup>2</sup>)</p> <p>1 square meter (m<sup>2</sup>) = 1.2 square yards (sq yd, yd<sup>2</sup>)</p> <p>1 square kilometer (km<sup>2</sup>) = 0.4 square mile (sq mi, mi<sup>2</sup>)</p> <p>10,000 square meters (m<sup>2</sup>) = 1 hectare (ha) = 2.5 acres</p>
<p><b>MASS - WEIGHT (APPROXIMATE)</b></p> <p>1 ounce (oz) = 28 grams (gm)</p> <p>1 pound (lb) = 0.45 kilogram (kg)</p> <p>1 short ton = 2,000 pounds (lb) = 0.9 tonne (t)</p>	<p><b>MASS - WEIGHT (APPROXIMATE)</b></p> <p>1 gram (gm) = 0.036 ounce (oz)</p> <p>1 kilogram (kg) = 2.2 pounds (lb)</p> <p>1 tonne (t) = 1,000 kilograms (kg) = 1.1 short tons</p>
<p><b>VOLUME (APPROXIMATE)</b></p> <p>1 teaspoon (tsp) = 5 milliliters (ml)</p> <p>1 tablespoon (tbsp) = 15 milliliters (ml)</p> <p>1 fluid ounce (fl oz) = 30 milliliters (ml)</p> <p>1 cup (c) = 0.24 liter (l)</p> <p>1 pint (pt) = 0.47 liter (l)</p> <p>1 quart (qt) = 0.96 liter (l)</p> <p>1 gallon (gal) = 3.8 liters (l)</p> <p>1 cubic foot (cu ft, ft<sup>3</sup>) = 0.03 cubic meter (m<sup>3</sup>)</p> <p>1 cubic yard (cu yd, yd<sup>3</sup>) = 0.76 cubic meter (m<sup>3</sup>)</p>	<p><b>VOLUME (APPROXIMATE)</b></p> <p>1 milliliter (ml) = 0.03 fluid ounce (fl oz)</p> <p>1 liter (l) = 2.1 pints (pt)</p> <p>1 liter (l) = 1.06 quarts (qt)</p> <p>1 liter (l) = 0.26 gallon (gal)</p> <p>1 cubic meter (m<sup>3</sup>) = 36 cubic feet (cu ft, ft<sup>3</sup>)</p> <p>1 cubic meter (m<sup>3</sup>) = 1.3 cubic yards (cu yd, yd<sup>3</sup>)</p>
<p><b>TEMPERATURE (EXACT)</b></p> <p><math>[(x-32)(5/9)] \text{ } ^\circ\text{F} = y \text{ } ^\circ\text{C}</math></p>	<p><b>TEMPERATURE (EXACT)</b></p> <p><math>[(9/5)y + 32] \text{ } ^\circ\text{C} = x \text{ } ^\circ\text{F}</math></p>

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## Executive Summary

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The Federal Railroad Administration (FRA) Office of Research and Development used signal detection theory (SDT) to model motorist stopping behavior at active highway-rail grade crossings (i.e. those equipped with active warning devices such as flashing lights and train-activated gate arms). The best fitting model indicated that mean expected train arrival time is not important, but that expected train arrival time variability (variance) determines stopping behavior. The model predicts that a motorist who is approaching an activated crossing is more likely to proceed if his or her experience is that trains arrive between, 19 and 21 seconds, compared to a variation of between 10 and 30 seconds. In other words, increasing train arrival time variability increases the tendency of motorists to stop. If this model is confirmed, engineering research efforts should be focused on maximizing train arrival time variance. More empirical information about motorist behavior and waiting time is needed to definitively confirm the model.

This report proposes a predictive model of motorist behavior at active grade crossings that helps explain how long motorists will stop at a grade crossing before driving across the railroad tracks. Many grade crossing collisions involve motorists who drive around gates or proceed across railroad tracks when flashing lights indicate that a train is approaching. Noncompliance with advance warning signs and devices has been linked to motorists' expectations of train arrival time following the activation of gates and lights and to the overall duration of the warning. Motorists' expectation of train arrival time following warning activation is important in motorists' decisions to abide by such warnings and stop at a grade crossing. However, it is not clear whether mean expected arrival time, variance in expected arrival time, or both influence stopping behavior.

Within the framework of SDT, expected train arrival time was modeled with Gaussian, Chi-squared and Poisson probability distributions. The Gaussian model, which is the only model in which the mean and variance are independent, provided the best fit to the motorist compliance data from Richards and Heathington (1990). In the Gaussian model, only train arrival time variance accurately determined stopping behavior. While this suggests that efforts to minimize train arrival time variance are important, the models have only been tested against limited data. Consequently, there is a need for more information about motorist behavior and train arrival time. Field studies, simulator studies and behavior simulations can provide valuable data to test and refine the model.

This report demonstrates the importance of developing a theory on motorist behavior at grade crossings. A theory of motorist behavior at grade crossings, such as SDT, provides a means to critically examine facts and inchoate hypotheses so that they can be more formally stated and vigorously tested. Theory allows facts to be organized in a meaningful and insightful way that expands understanding of the issues and leads to the generation of new hypotheses.

# 1. Introduction

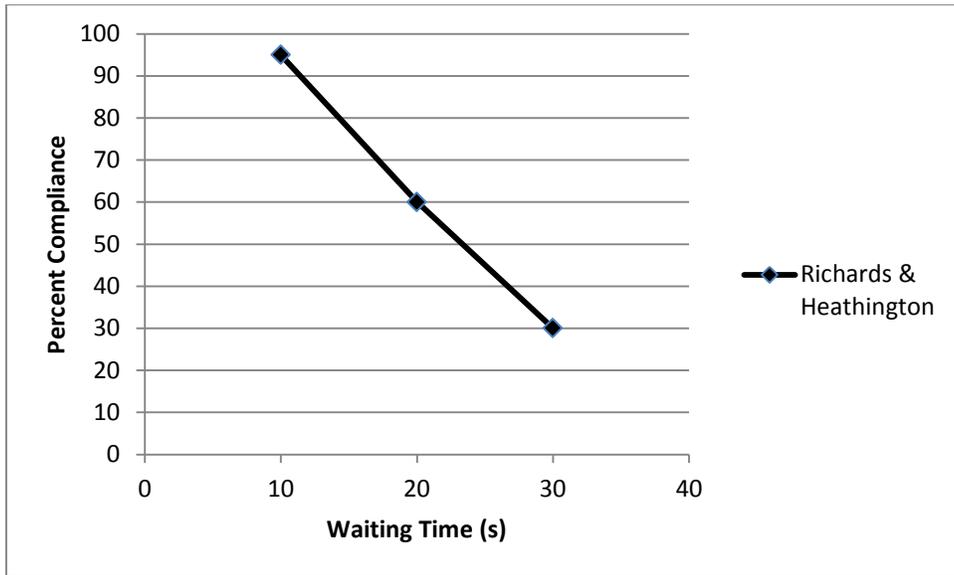
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Motorist error or poor judgment is considered a significant factor in grade crossing collisions (Office of the Inspector General, 2004). A recent Federal Railroad Administration report (Yeh, Raslear and Multer, 2013) specifically notes that motorist expectations of long waiting times at grade crossings is frequently used as a hypothesis to account for motorist noncompliance at grade crossings.

This report proposes a model of motorist behavior at grade crossings which explains how long motorists will stop at a grade crossing before driving across the tracks. This is an important issue, because most grade crossing collisions involve motorists who drive around gates or proceed across railroad tracks when flashing lights indicate that a train is approaching. This behavior has been linked to motorists' expectations of train arrival time following the activation of gates and lights as well as the overall duration of the warning (see Yeh and Multer (2008) for details). Yeh and Multer note that the Manual of Uniform Traffic Control Devices (MUTCD) and FRA regulations require a minimal advance warning time of 20 s. However, warning times vary considerably because the track circuits that activate crossing gates and other warning signals are often at a fixed distance from the grade crossing as well as other contextual variables (i.e. the geometry of crossing, the number of tracks, the mix of freight/passenger traffic, train frequencies, among others). Consequently, variations in train speed cause variability in warning times and increases in warning times have been linked to increases in grade crossing violations.

In Yeh and Multer's review of the grade crossing safety literature on warning times, they cite a report by Richards and Heathington (1990) which states that motorists expect a train within 20 s of warning device onset. 95 percent of motorists stop and wait within 10 s of train arrival, more than 50 percent stop and wait within 10 to 20 s of train arrival, and 30 percent stop and wait within more than 20 s of train arrival (see Figure 1). Many researchers seem to view the variability in warning time as the main contributor to warning times which are longer than motorists' expectations, which result in violations and accidents. This, however, conflates the variability (variance,  $\sigma^2$ ) of the time that the train arrives with the train's mean time to arrival ( $\mu$ ). It is not clear whether  $\mu$ ,  $\sigma^2$  or both contribute to motorist noncompliance or to what degree. Motorist expectations about train arrival could depend on  $\mu$ ,  $\sigma^2$  regardless of  $\mu$ , or  $\sigma^2$  relative to  $\mu$ .

In the grade crossing safety literature, variability in train arrival time has been viewed as a likely cause of motorist non-compliance *per se*. For instance, Ogden (2007, p. 128) notes that "Reasonable and consistent warning times reinforce system credibility. Unreasonable or inconsistent warning times may encourage undesirable driver behavior." Jenness et al. (2006, p. 130) further state that "Drivers must perceive train warning information to be accurate and reliable. Predictions of train arrival time should be as accurate as possible," and they also say that "Inconsistency in operational aspects affects the road user's ability to comply through its effect on driver expectancy. Road users will have some expectancy about probable messages and message formats, such as where the message should be located, how it is timed, how much time they have to respond, and so forth" (p. 56). It is implicit that variability in all aspects of warnings is undesirable, although there is scant research on the effects of variability for train warnings. This report directly explores that issue through the SDT model.



**Figure 1. Percent motorist compliance as a function of waiting time. Based on Richards and Heathington, 1990.**

## 2. The Model

---

Motorist expectation has been previously modeled with Signal Detection Theory (SDT, see Raslear, 1996; Yeh and Multer, 2008; Yeh *et al.*, 2013). Expectation in SDT is generally reflected in bias ( $\beta$ , see McNicol, 1972, p. 9), where

$$\beta = \frac{p(\text{no train})}{p(\text{train})} \frac{V(CC)+V(FS)}{V(VS)+V(AC)} \quad (1)$$

The value of  $\beta$  is determined by the prior odds of a train being close [ $\frac{p(\text{no train})}{p(\text{train})}$ ] and the value function for the decision outcomes [ $\frac{V(CC)+V(FS)}{V(VS)+V(AC)}$ ]. Prior odds are defined here as the probability ( $p$ ) of a null event relative to the probability of the event. In SDT, as applied to motorist behavior at grade crossings, there are two states of the world: a train is close, or a train is not close. Motorists decide to either stop or proceed. The decision outcomes are shown in Table 1.  $V(CC)$ ,  $V(FS)$ ,  $V(VS)$  and  $V(AC)$  are the values ( $V$ ) associated with each of the four decision outcomes shown in Table 1. For example,  $V(AC)$  might be the cost of new car if the highway vehicle is damaged beyond repair,  $V(FS)$  might be the cost of arriving late to work,  $V(CC)$  might be the benefit accruing to safely crossing the tracks without delay, and  $V(VS)$  might be the benefit associated with avoiding an accident. These costs and benefits are often subjective and difficult to quantify. Consequently, we consider the prior odds to be the principal determinant of motorist expectation, and we set the value function equal to 1.

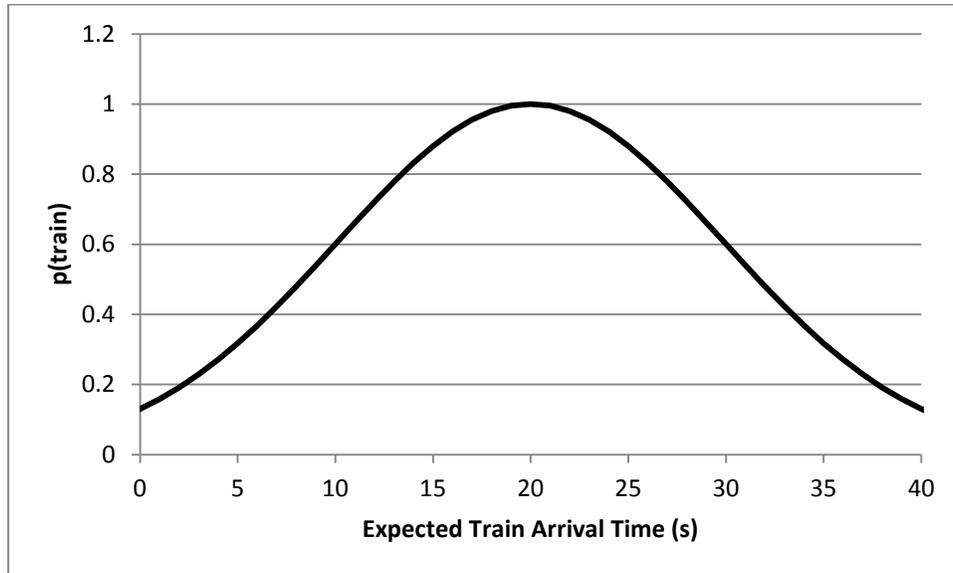
**Table 1. Decision outcomes for a motorist at a grade crossing.**

	<u>DECISION</u>	
	STOP	PROCEED
<u>STATE OF THE WORLD</u>		
<b>TRAIN IS CLOSE</b>	Valid Stop (VS)	Accident (AC)
<b>TRAIN NOT CLOSE</b>	False Stop (FS)	Correct Crossing (CC)

The *subjective* probability of a train in the grade crossing can be represented by a Gaussian-like function as shown in Figure 2. Figure 2 has the familiar bell-shaped form of a normal probability density function (PDF), but is not a PDF since its integral would be greater than 1 (in fact the value of  $p(\text{train})$  at 20 s is 1). The equation for this function is

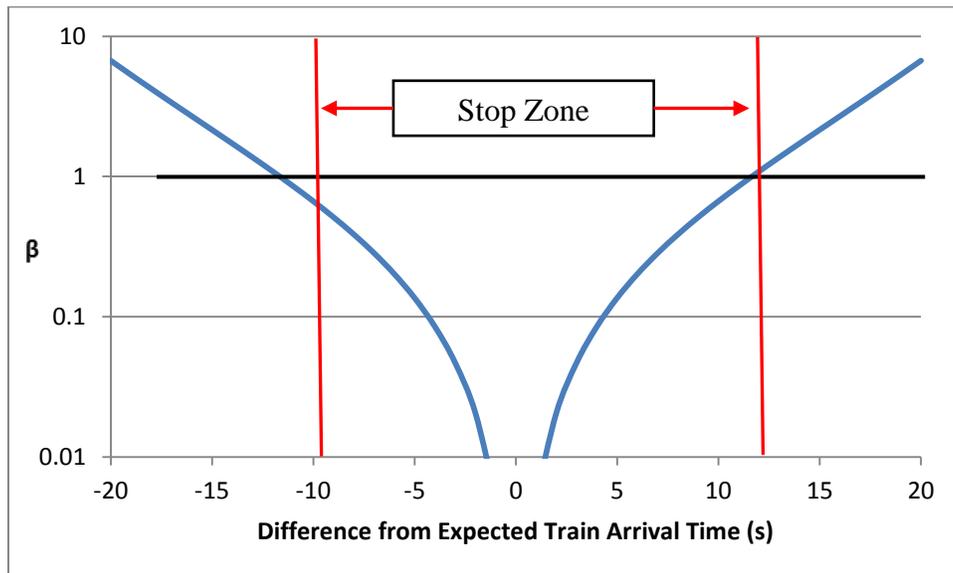
$$p(\text{train}) = \exp\left[-\frac{(t-MTA)^2}{2\sigma^2}\right]. \quad (2)$$

Equation (2) differs from the Gaussian PDF by a factor of  $\frac{1}{\sigma(2\pi)^{0.5}}$ .  $\mu$  in Figure 2 is 20 s, and  $\sigma = 7$  s.



**Figure 2. Subjective probability of a train in the grade crossing as a function of expected train arrival time.**

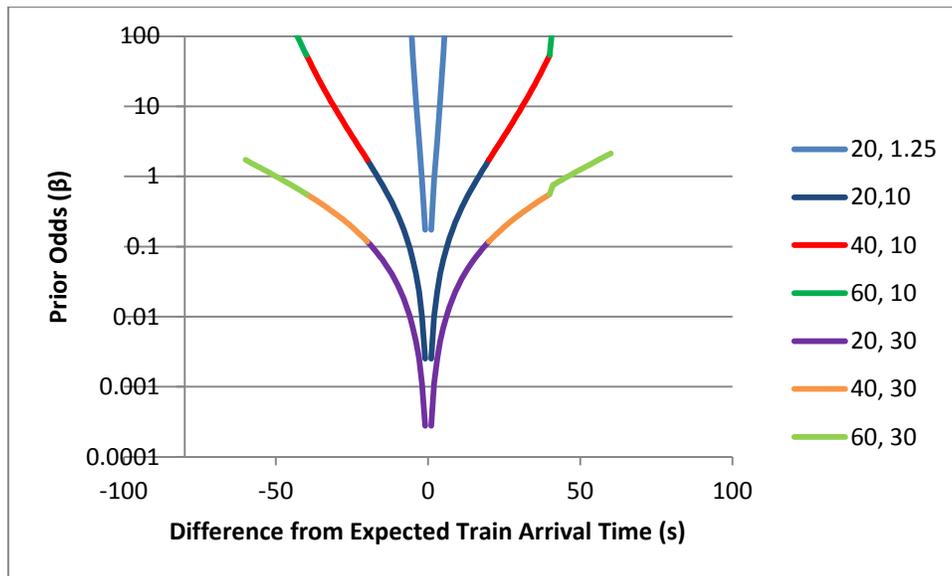
$\beta$  (actually the prior odds since the value function is set to 1), based on the function in Figure 2 and Equation (1), is shown in Figure 3.



**Figure 3.  $\beta$  as a function of difference from expected train arrival time. The Stop Zone indicates the time interval during which motorists are likely to stop.**

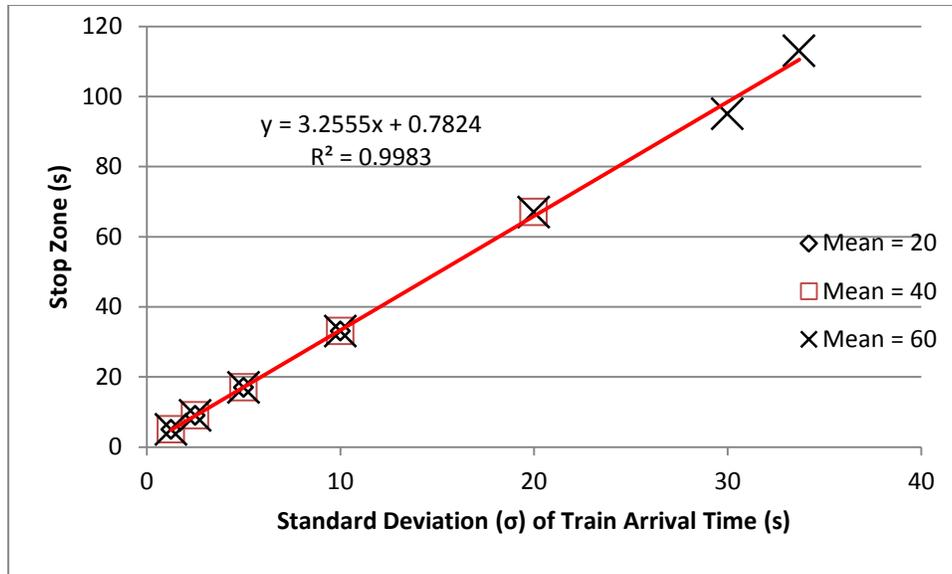
When  $\beta > 1$ , there is a bias to proceed. When  $\beta < 1$ , there is a bias to stop. At  $t = -20$  s, warning signals at active grade crossings are activated. Figure 3 shows that there is a bias to proceed at  $t < -11.5$  s (8.5 s after the warning signals are activated) and at  $t > 11.5$  s (31.5 s after the warning signals are activated). In other words, the Stop Zone for  $\mu = 20$ s,  $\sigma = 7$  s is 8.5 – 31.5 s. The greatest bias to stop is at  $\mu$  s ( $t = 0$ ).

The prior odds as a function of  $\mu$  and  $\sigma$  are shown in Figure 4. While there is obviously an effect of both  $\mu$  and  $\sigma$ , the Stop Zone (the point where the functions intersect 1), is much more dependent on  $\sigma$ . This can be seen more clearly in Figure 5, which shows the Stop Zone width as a function of  $\sigma$  and  $\mu$ .



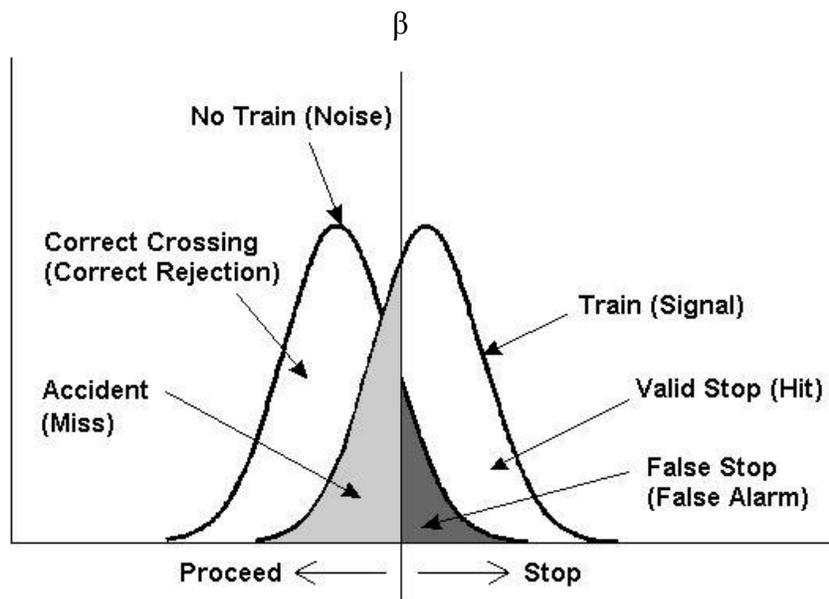
**Figure 4. Prior odds as a function of  $\mu$  and  $\sigma$ .  $\mu$ s of 20, 40 and 60 s are shown with  $\sigma$ s of 1.25, 10 and 30 s.**

Figure 5 shows that  $\sigma$  accounts for almost 100% of the variance in the Stop Zone width. The width of the Stop Zone at each  $\sigma$  is the same regardless of the value of  $\mu$ . This is consistent with the property of the Gaussian distribution that the mean is independent of the standard deviation.



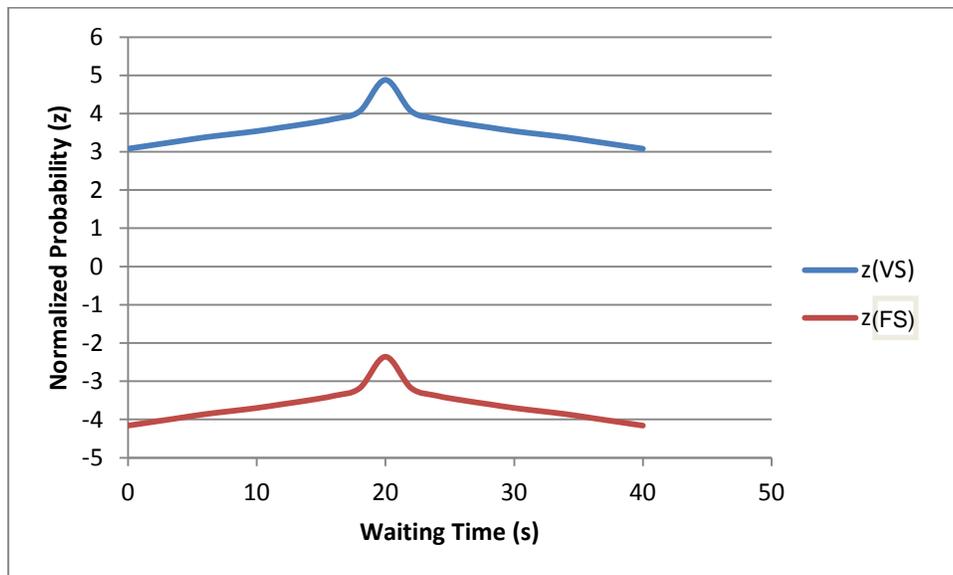
**Figure 5. The Stop Zone width as a function of MTA and  $\sigma$ .**

Because  $\beta$  is a function of time, decision outcomes, such as the probability of an accident ( $p(AC)$ ) or valid stop ( $p(VS)$ ), also change as a function of time, which means that this is a dynamic model. This is illustrated in Figure 6, which shows the relationship between  $\beta$  and the decision outcomes. When  $\beta$  shifts to the left (decreases), the bias to stop increases. As a result,  $p(AC)$  and  $p(CC)$  decrease, while  $p(VS)$  and  $p(FS)$  increase.



**Figure 6. The underlying distributions for No Train and Train and the relationship of  $\beta$  to the probabilities of decision outcomes.**

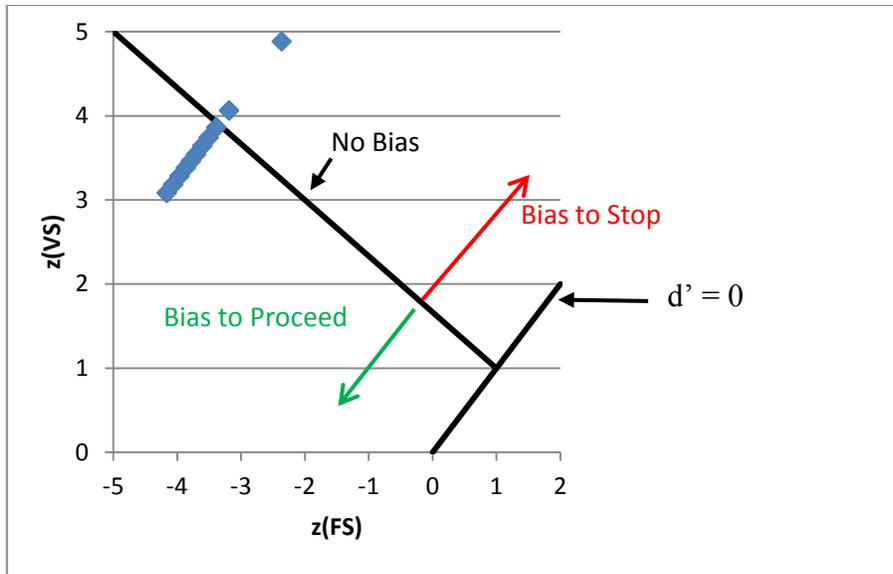
To explore the dynamics of the model, Gaussian signal and noise distributions were created in an Excel spreadsheet. For the noise distribution, mean = 10, standard deviation = 5 (arbitrary units since we are modeling the perception of the train, not the subjective time of arrival of the train); for the signal distribution, mean = 45.7, standard deviation = 5. These values were chosen, based on previous SDT analyses of motorist behavior at active grade crossings, to set  $d' = 7.14^1$  (Yeh, et al., 2013). The distribution for expected train arrival time had  $\mu = 20$  s and  $\sigma = 7$  s using Equation (2). At  $t = 0$  in Equation (2), the magnitude of signal and noise was set equal to zero. Signal and noise magnitudes increased by 0.1 units with each increase of 0.1 s in  $t$ .<sup>2</sup> Figure 7 shows the changes in normalized Gaussian probability for Valid Stops,  $z(\text{VS})$ , and for False Stops,  $z(\text{FS})$ , as a function of waiting time. The difference between the curves is always 7.14 units since  $d' = 7.14$ . A peak in both curves occurs at  $t = 20$  s. Figure 8 shows the traditional SDT receiver operating characteristic (ROC) plot of  $z(\text{VS})$  vs.  $z(\text{FS})$  using the same data.



**Figure 7.  $z(\text{VS})$  and  $z(\text{FS})$  as a function of waiting time.**

<sup>1</sup>  $d'$  is a measure of sensitivity in SDT. It is the distance between the peaks (means) of the signal and noise distributions in Fig. 6 in standard deviation units ( $d' = 7.14 = \frac{45.7-10}{5}$ ). High values of  $d'$ , such as 7.14, indicate that a train is highly detectable.

<sup>2</sup> This mapping of expected train arrival time to signal and noise magnitudes is arbitrary and for purposes of illustration only. An empirical study would be necessary for an actual mapping.

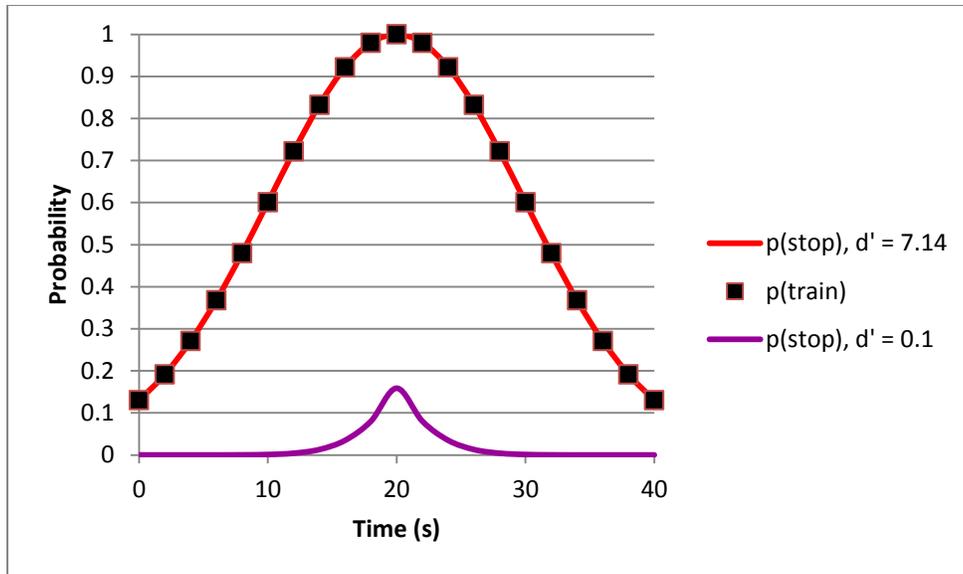


**Figure 8. ROC plot of data from Figure 7.**

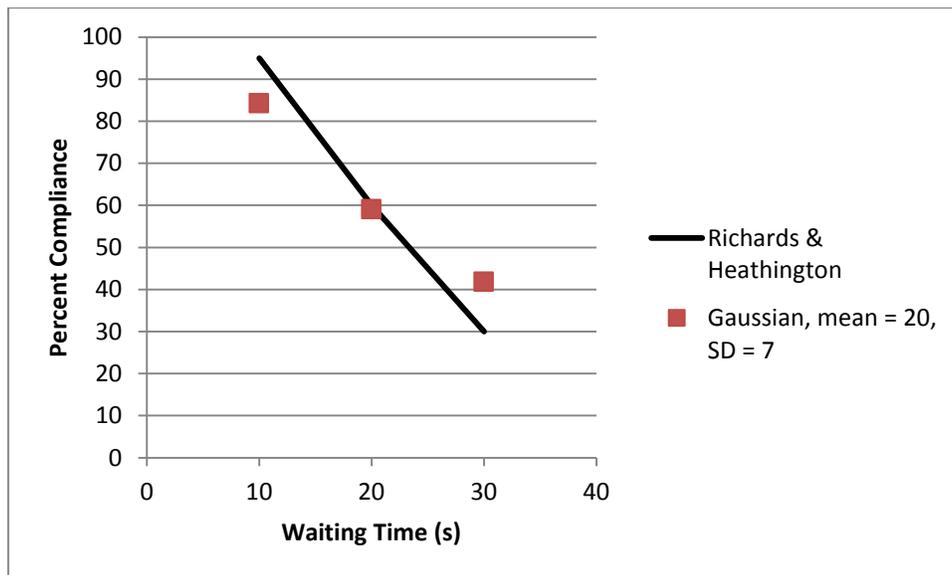
The probability of stopping (see Egan, 1975, p. 9) is

$$p(\text{Stop}) = [p(VS) \cdot p(\text{train})] + [p(FS) \cdot (1 - p(\text{train}))]. \quad (3)$$

Figure 9 plots  $p(\text{Stop})$  and  $p(\text{train})$  as a function of expected train arrival time and train detectability.  $p(\text{Stop})$  is almost identical to  $p(\text{train})$  when  $d' = 7.14$  ( $r^2 = .9999$ ). However, under conditions of low sensitivity ( $d' = 0.1$ ),  $p(\text{Stop})$  is moderately predicted by  $p(\text{train})$  ( $r^2 = .256$ ), and the functions are quite distinct. Obviously, detectability of the train is quite important. The implication is that if the train is not highly visible because it is far away or there is an obstructed view, motorists are less likely to stop. This is consistent with SDT in which sensitivity and bias are independent in determining decision outcomes.



**Figure 9.  $p(\text{Stop})$  and  $p(\text{train})$  as a function of expected train arrival time with  $d' = 7.14$  and  $0.1$ .**



**Figure 10. Percent motorist compliance as a function of waiting time. The data of Richards and Heathington are modeled by a Gaussian function.**

Figure 10 shows that the calculations of  $p(\text{stop})$ , or compliance, based on a Gaussian model of motorist expectation of train arrival ( $\mu=20\text{s}$ ,  $\sigma = 7\text{s}$ ) and high train detectability ( $d' = 7.14$ ), approximates the Richards and Heathington data very well ( $r^2 = 0.9959$ ) even though no attempt was made to specifically fit the data. A  $\chi^2$  test for goodness of fit (Hays, 1963) indicated that the Gaussian model was an adequate fit to the data ( $\chi^2 = 5.89$ ,  $df = 2$ , critical value = 5.99,  $p > 0.05$ ). In Figure 10, the values for percent compliance are averaged over the ranges that are indicated

by the Richards and Heathington report: the value for “within 10 s of the train arrival” is the average of  $p(\text{stop}) \times 100$  from 10 to 20 s, the value for “within 20 s of the train arrival” is the average of  $p(\text{stop}) \times 100$  from 0 to 20 s, and the value for “within 30 s of the train arrival” is the average of  $p(\text{stop}) \times 100$  from -10 to 20 s.

### 3. Expected Train Arrival Time Using Other Probability Distributions

#### 3.1 The Chi-Squared Distribution

The Gaussian distribution is convenient to use and appears to adequately model what we know about stopping behavior at active grade crossing devices. There are, however, other distributions that can be used. Since both the Chi-Squared ( $\chi^2$ ) and the Poisson distributions have been used to model time perception, they are of interest (Creelman, 1962; Gibbon, 1977). Unlike the Gaussian distribution, the mean and standard deviation are related in the  $\chi^2$  distribution. The mean of the  $\chi^2$  is equal to the number of degrees of freedom ( $\nu$ ), and the standard deviation is  $\sqrt{2\nu}$ . The equation for  $\chi^2$  is

$$\chi^2 = \frac{x^{(\nu-2)/2} \exp(-\frac{x}{2})}{2^{\nu/2} \Gamma(\frac{\nu}{2})}, \quad (4)$$

where  $\Gamma$  is the gamma function.

Figure 11 shows the subjective probability of train arrival time using the  $\chi^2$  distribution with  $\nu = 21$ . Probabilities have been multiplied by a constant so that peak of the function has a probability of 1. Note that the function has a peak at  $t = 20$  s in accordance with the data (as in Figure 2). Unlike the Gaussian distribution, the  $\chi^2$  is asymmetric and this results in an asymmetric function for the prior odds (as in Figure 12). The Stop Zone in this case is 13.5 – 27.5 s ( $t < -6.5$  s to  $t > 7.5$  s). The Stop Zone for  $\chi^2$  differs from the Gaussian in two ways: the zone is shorter in duration (14 s vs. 23 s) and is asymmetric (the stopping duration is 1 s longer at  $t > 0$ ).

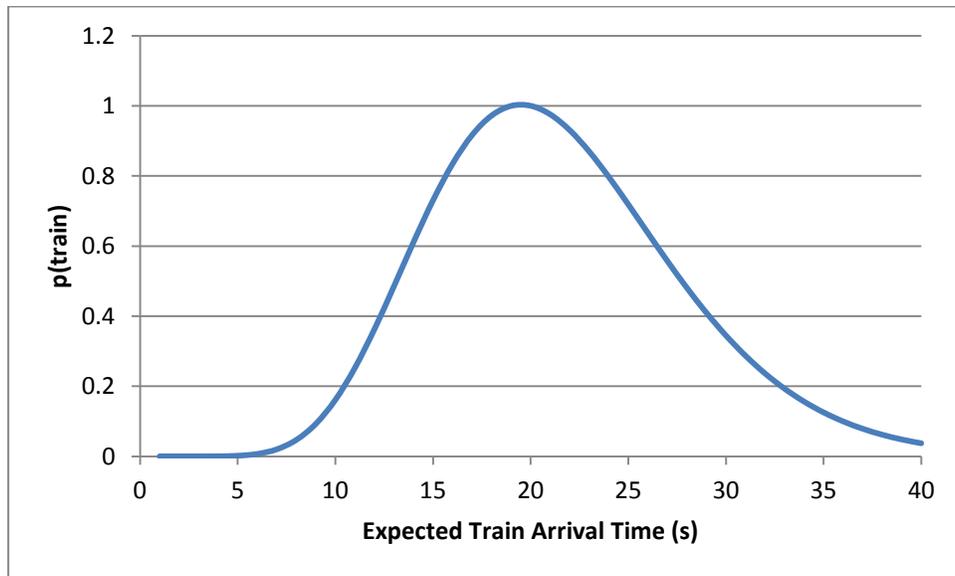
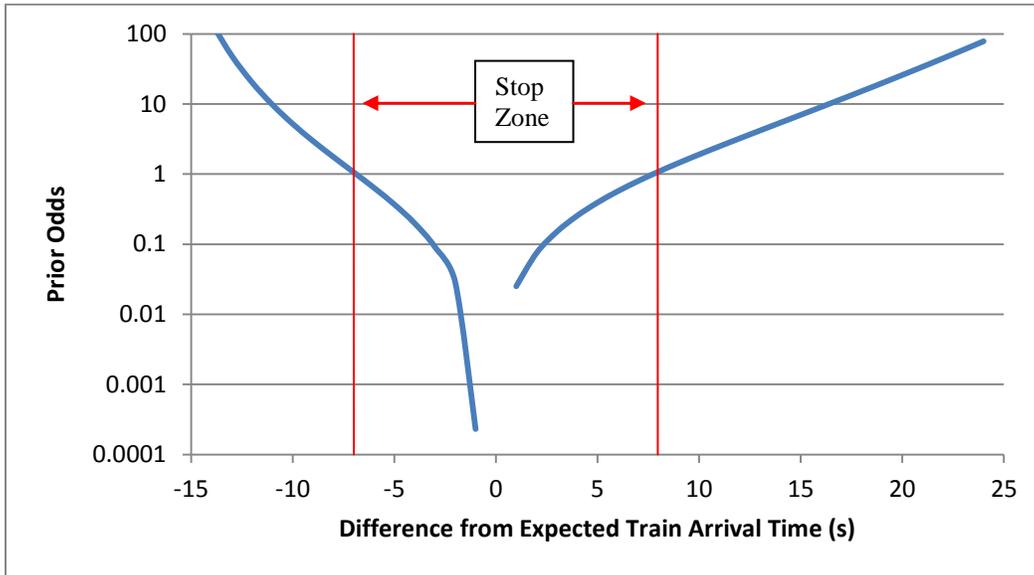
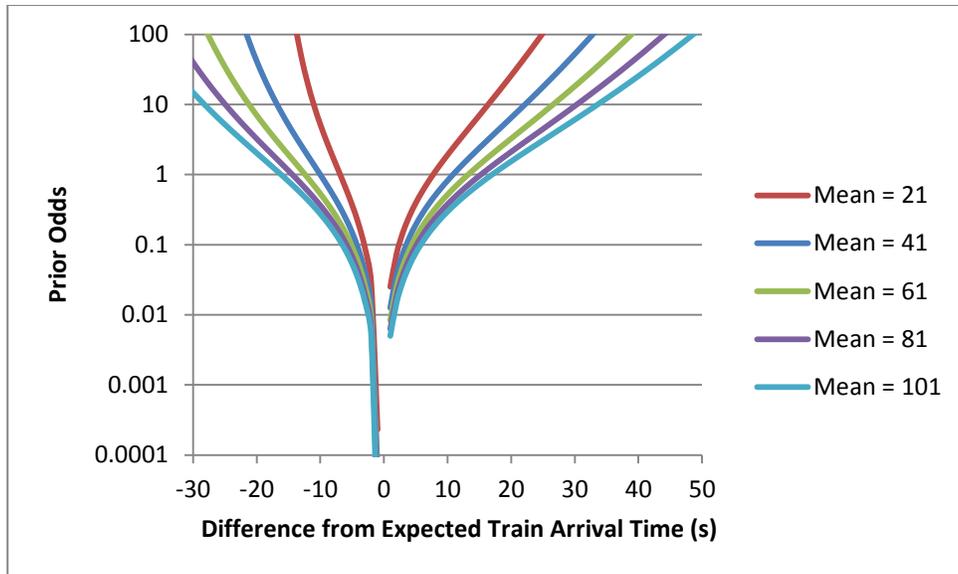


Figure 11. Subjective probability of train arrival time using the  $\chi^2$  distribution with  $\nu = 21$ .

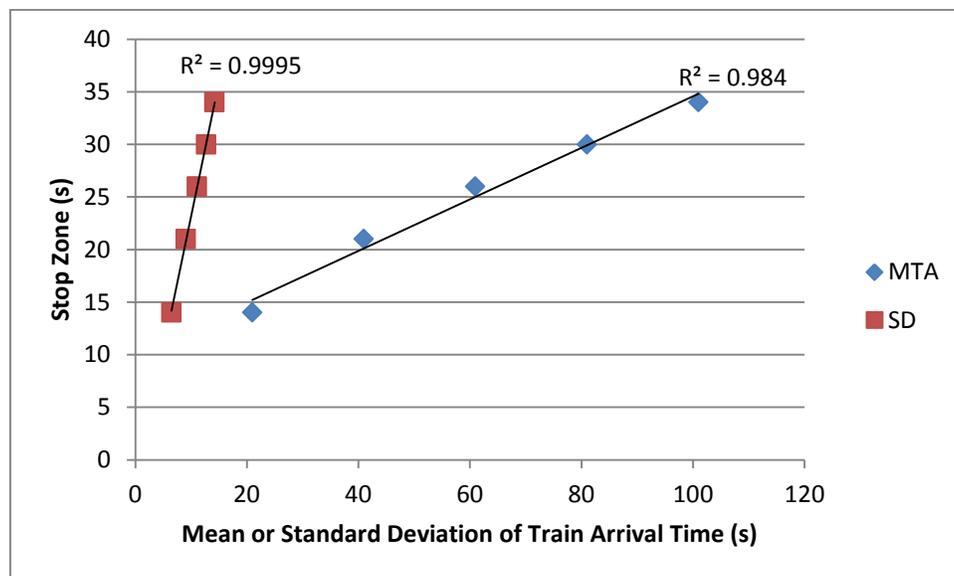


**Figure 12. Prior odds as a function of difference from expected train arrival time. The  $\chi^2$  probabilities in Figure 10 were used to calculate the prior odds. The Stop Zone indicates the time during which motorists are likely to stop.**

The prior odds as a function of the mean ( $\nu$ ) is shown in Figure 13. Five values of  $\nu$  were examined to provide a comparison with Figure 4. The shape of the function in all cases is asymmetric and it is clear that the prior odds are a function of  $\nu$ . Figure 14 shows the Stop Zone as a function of the mean and the standard deviation of the functions in Figure 13. Since the mean and standard deviation are related, it is not surprising that both are predictive of the Stop Zone.  $p(\text{stop})$  for the  $\chi^2$  model resembles the pattern seen for the Gaussian model in Figure 9 (not shown).

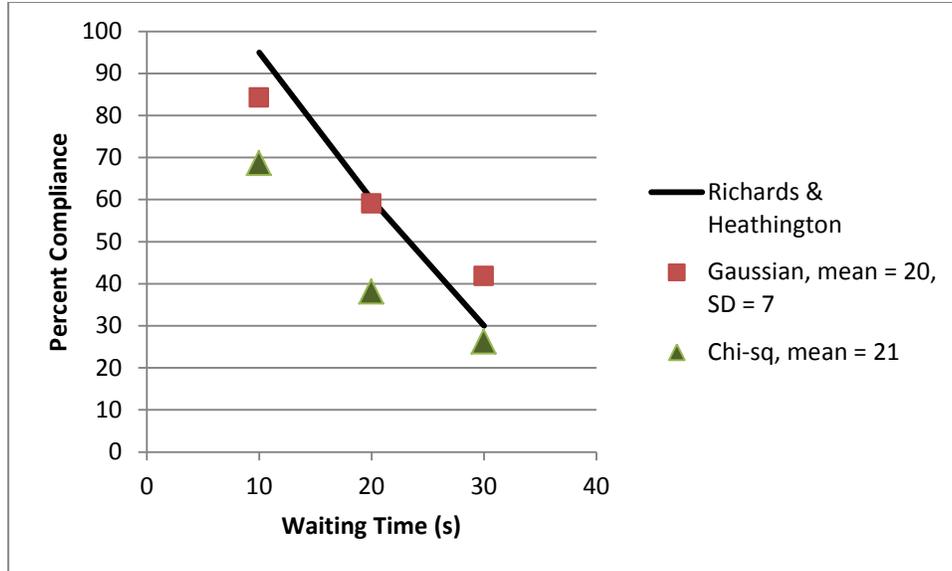


**Figure 13. Prior odds as a function of the mean ( $\nu$ ) of the  $\chi^2$  distribution.**



**Figure 14. The Stop Zone as a function of the mean and standard deviation for  $\chi^2$  distributions.**

Using a different distribution predicts different motorist behavior. Figure 15 shows the predicted compliance (stopping) for this model relative to the Richards and Heathington data and the Gaussian model. As was the case for the Gaussian model, train detectability is high ( $d' = 7.14$ ), and the values for percent compliance are averaged over the same ranges as indicated by the Richards and Heathington report.



**Figure 15. Percent motorist compliance as a function of waiting time for  $\chi^2$  and Gaussian models compared with empirical data from Richards and Heathington.**

It is clear in Figure 15 that the  $\chi^2$  model does not do as well as the Gaussian model. Although there is a high correlation between the empirical data and the prediction ( $r^2=0.959$ ), the model underestimates the data by 17.3 percent on the average. By comparison the Gaussian model overestimates the data by 0.05 percent. A  $\chi^2$  test for goodness of fit (Hays, 1963) indicated that the  $\chi^2$  model was not a good fit to the data ( $\chi^2 = 15.71$ ,  $df=2$ , critical value = 5.99,  $p < 0.05$ ).

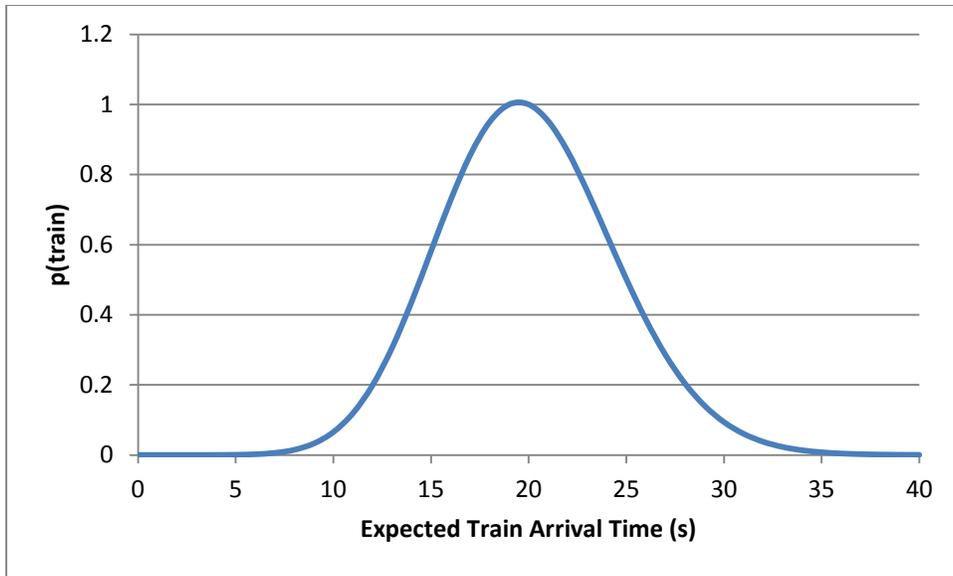
### 3.2 The Poisson Distribution

The Poisson distribution is also used to model time perception (as noted above). Like the  $\chi^2$ , the mean and standard deviation are related. For the Poisson, the mean is denoted by  $\lambda$  and the standard deviation is equal to  $\sqrt{\lambda}$ . Poisson probabilities are given by

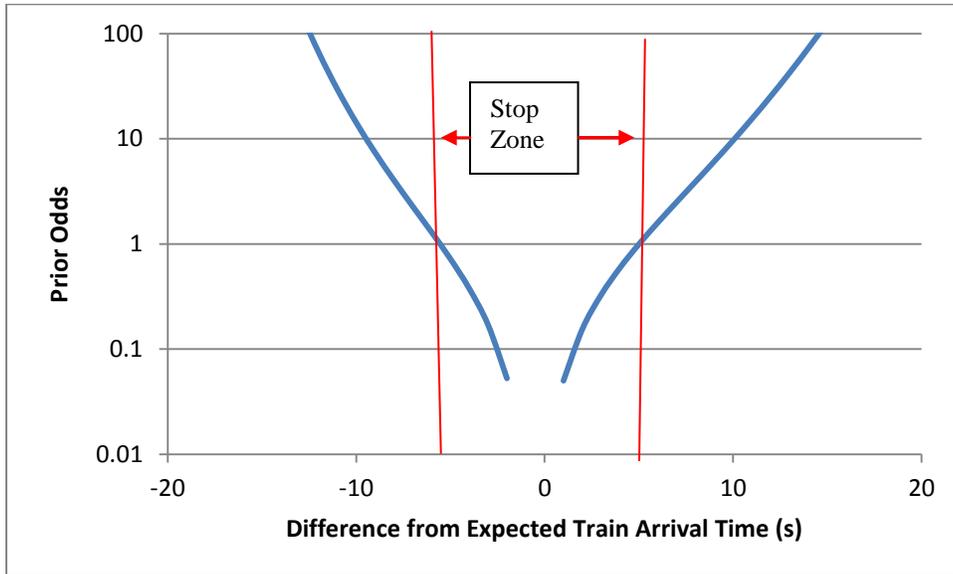
$$\frac{e^{-\lambda}\lambda^x}{x!}, \quad (5)$$

where  $x$  is the number of occurrences of a random event in an interval of time.  $\lambda$  is the average number of events in the interval.

Figure 16 shows the subjective probability of train arrival time using the Poisson distribution with  $\lambda = 20$ . Probabilities have been multiplied by a constant so that the peak of the function has a probability of 1. Note that the function has a peak at  $t = 20$  s (as in Figure 2 and Figure 11) to conform with the motorist expectation data that a train will arrive within 20 s of warning device onset. The shape of the function is asymmetric, like that for the  $\chi^2$  model, although this is harder to discern: the right tail of the distribution is heavier. This can be more clearly seen in the asymmetry of the prior odds which is shown in Fig. 17. The Stop Zone for the Poisson model with  $\lambda = 20$  is 14.5 - 25.5 s ( $t < -5.5$  s to  $t > 5.5$  s). This Stop Zone is shorter in duration (11 s) than the  $\chi^2$  model (14 s) or the Gaussian model (23 s).

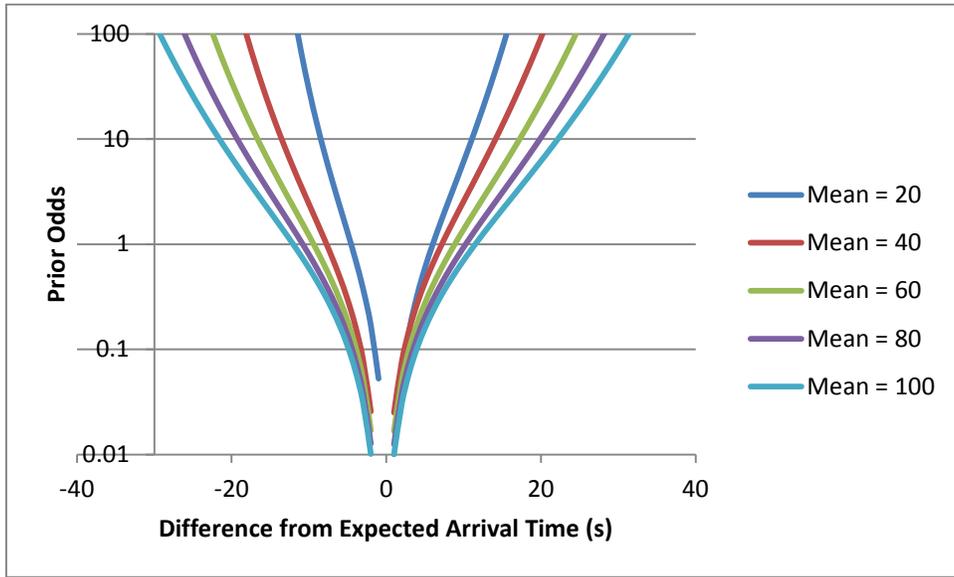


**Figure 16. Subjective probability of train arrival time using the Poisson distribution with  $\lambda = 20$ .**

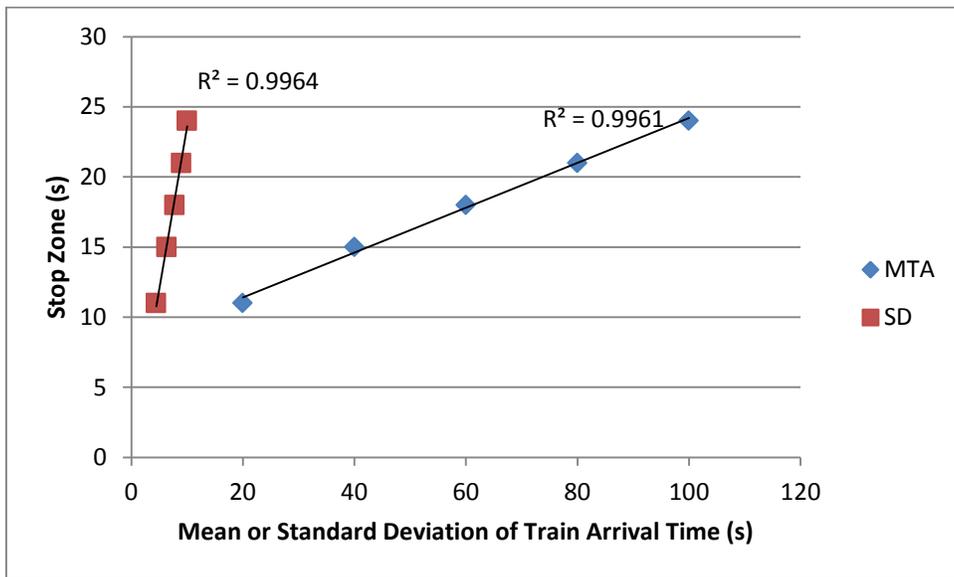


**Figure 17. Prior odds as a function of difference from expected train arrival time. The Poisson probabilities in Figure 16 were used to calculate the prior odds. The Stop Zone indicates the time during which motorists are likely to stop.**

The prior odds as a function of  $\lambda$  is shown in Figure 18. Five values of  $\lambda$  were examined to provide a comparison with Figure 4 and Figure 13. The shape of the function becomes less asymmetric as  $\lambda$  increases, and it is clear that the prior odds are a function of  $\lambda$ . Figure 19 shows the Stop Zone as a function of the mean ( $\lambda$ ) and standard deviation ( $\lambda^{1/2}$ ) of the functions in Figure 18. As was the case for the  $\chi^2$  model, the mean and the standard deviation are both predictive of the Stop Zone, and  $p(\text{stop})$  for the Poisson model resembles the pattern seen for the Gaussian model in Figure 9 (not shown).



**Figure 18. Prior odds as a function of the mean ( $\lambda$ ) of the Poisson distribution.**

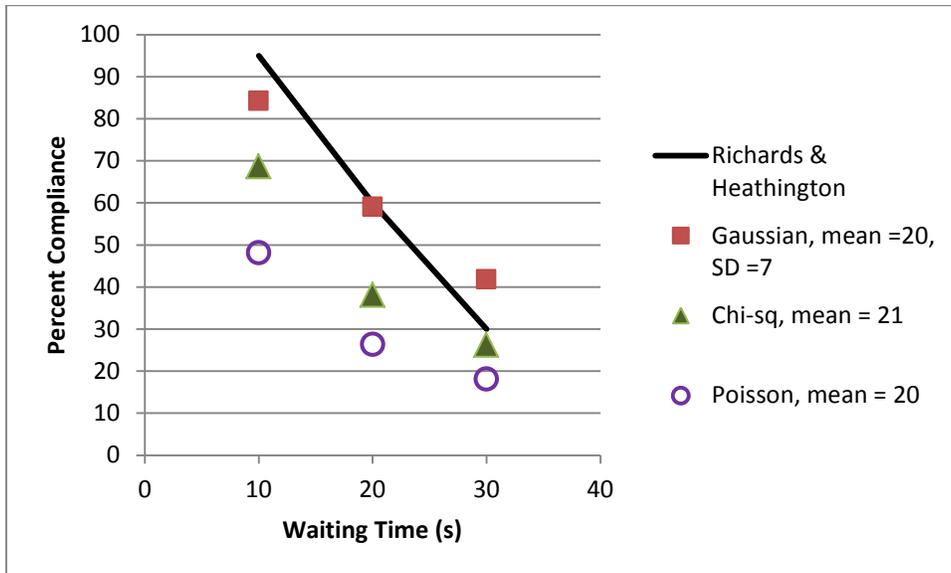


**Figure 19. The Stop Zone as a function of the mean and standard deviation for Poisson distributions.**

Figure 20 shows the predicted compliance (stopping) for the Poisson model relative to the Richards and Heathington data, the Gaussian model and the  $\chi^2$  model. As was the case for the Gaussian and  $\chi^2$  models, train detectability is high ( $d'=7.14$ ), and the values for percent compliance are averaged over the same ranges as indicated by the Richards and Heathington report.

The Poisson model does not do as well as the Gaussian or  $\chi^2$  models. Although there is a high correlation between the empirical data and the prediction ( $r^2=0.956$ ), the Poisson model

underestimates the data by 30.7 percent on the average. By comparison the Gaussian model overestimates the data by 0.05 percent and the  $\chi^2$  model underestimates the data by 17.3 percent. A  $\chi^2$  test for goodness of fit (Hays, 1963) indicated that the Poisson model was not a good fit to the data ( $\chi^2 = 46.59$ ,  $df = 2$ , critical value = 5.99,  $p < 0.05$ ).



**Figure 20. Percent motorist compliance as a function of waiting time for the Poisson,  $\chi^2$  and Gaussian models compared with empirical data from Richards and Heathington.**

## 4. Discussion and Next Steps

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It is clear that SDT can be used to understand motorist stopping behavior at active grade crossings. Examination of three probability distributions to describe stopping behavior suggests that motorist expectation about train arrival may follow a Gaussian model since that model provides an acceptable fit to the Richards and Heathington data. In the Gaussian model mean train arrival time is not important, but train arrival time variance determines stopping behavior (see Figure 5) in that the stop zone increases with increases in the train arrival time variance. This seems counter-intuitive since variability of train arrival time at grade crossings is often thought to be the cause of motorist non-compliance. It is implicit in the grade crossing literature that increased variability in train arrival time, *per se*, (Jenness et al., 2006; Ogden, 2007) results in undesirable motorist behavior (e.g., non-compliance). However, the model makes sense if motorists are sensitive to uncertainty regarding train arrival. Under conditions with greater uncertainty about the arrival time of the train (higher variance or variability), it makes sense for a motorist to remain stopped. If the model is confirmed, engineering research efforts should be funded to maximize train arrival time variance. At this time, however, such efforts would be premature because we are lacking sufficient empirical information about motorist behavior and train arrival time.

### 4.1 Data on Motorist Behavior

It is important to note that the Richards and Heathington data are not definitive. It has been useful to compare models against their data, but that comparison is only illustrative. Three data points from one study do not constitute a solid empirical description of motorist behavior.

More field data on motorist behavior at active warnings is needed. That data should include information about warning times relative to train arrival as well as motorist waiting time. Data should be collected at a variety of crossings with a variety of active warning devices. According to Yeh *et al.*, bias to stop varies by a factor of 25 between gates ( $\beta = 6.85E-05$ ) and special active warning devices ( $\beta = 0.0017$ ). The models we have described do not account for differences in bias to stop between different active devices (e.g., gates vs. special active warning devices). The Yeh *et al.* estimates of  $\beta$  are based on accident data, not driver behavior observations. Are there differences in stopping behavior across active warning devices? Devices differ with regard to daily train traffic. The probability of a train at any time in the day is a function of daily train traffic. The models we have described indicate the probability a train arriving at a time following a warning of train arrival. It should be possible to further refine these models to account for the conditional probability of a train arriving at a particular time given the warning of train arrival using Bayes' theorem.

We do not know what the relationship is between subjective train arrival time and actual train arrival time. The models use the distribution of subjective train arrival times to predict motorist stopping behavior, not actual train arrival time. Are there grade crossings which differ in regard to distribution of arrival times, mean arrival time, and arrival time variance? If these differences exist, do motorists who use these grade crossings have different subjective train arrival time distributions and different stopping behavior?

Data can be collected in the field or by using a simulator. Simulator studies can help define crossing characteristics that are important to include in more costly field studies. For instance, it

would be cost-effective to use a simulator to explore the relationship between stopping behavior and actual train arrival times. A factorially designed experiment to determine the influence of mean train arrival time and train arrival variance could be used to test the adequacy of the Gaussian model. A follow-up field study on the influence of train arrival time distributions on stopping behavior could be performed to validate the model.

## **4.2 Motorist Behavior Simulations**

As noted previously, the motorist behavior models are dynamic and can be used to simulate stopping behavior, subjective train arrival time, sensitivity and bias. Such simulations can be extremely useful for further refining and developing models of motorist behavior and decision making at grade crossings. Of course, motorist behavior simulations must be based on high quality data on motorist behavior at grade crossings.

## **4.3 The Role of Theory**

To this point in time, research on grade crossing accident causation and prevention has been largely empirical. Research in this area is often categorized by the Three “Es”: Engineering, Education and Enforcement. Such research tests a particular change in engineering design (e.g., stop signs instead of crossbucks), education (e.g., Operation Lifesaver, Inc.’s outreach to school children), or law enforcement (e.g., photo enforcement) to determine if the change results in a decrease in collisions or noncompliance. While this approach can advance safety, it does so in an unsystematic and disorganized way. Relationships between dependent and independent variables are not developed systematically and formalized as higher level explanatory concepts. Consequently, the research enterprise has no clear direction, and a unified view of the field is lacking. For instance, do we know what the relationship is between engineering, education and enforcement? Is one more important than the others? What aspects of grade crossing incidents require more attention to engineering, education or enforcement?

Theory provides a systematic way to view grade crossing accidents, allows generalizable new knowledge or insights to emerge (Leong, Schmitt and Lyons, 2012), and it is essential to the continued development of all fields. This report demonstrates the importance of developing theory on motorist behavior at grade crossings, such as SDT, which provide a means to critically examine inchoate hypotheses so that they can be more formally stated and vigorously tested. Theory organizes facts in a meaningful and insightful way that expands understanding of the issues and lead to the generation of new hypotheses. The current emphasis in grade crossing research on the three “Es”, Engineering, Education and Enforcement, is insufficient and does not substitute for theory.

#### **4.4 Conclusions**

Signal detection theory provides a useful basis for understanding motorist stopping behavior at active grade crossings. The literature on motorist behavior suggests that the difference between motorists' expectation of when a train will arrive at an active crossing and actual train arrival is the cause of noncompliance. The variability of train arrival time relative to active warnings is thought to drive the disparity between expectation and reality. The theoretical analysis in this report supports the hypothesis that train arrival time variability causes noncompliance. However, the data to support that conclusion is limited. More data on motorist stopping behavior at grade crossings is needed to test and refine the model suggested in this report. This report demonstrates the importance of theory in understanding grade crossing issues. The development of theory in this area should be emphasized.

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