Wheel Unloading of Rail Vehicles Due to Track Twist

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Cambridge, MA 02142

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Final Report

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The U.S. Government does not endorse products or manufacturers. Trade or manufacturers' names appear herein solely because they are considered essential to the objectives of this report.
An analysis is presented describing the effect that track twist has on the loads carried by the wheels of a rail car. Wheel unloading is determined as a function of the difference in crosslevel between the truck centers of the car. The different vehicle characteristics that affect a car's reaction to track twist are determined. It is found that light, torsionally stiff cars are the most susceptible to wheel unloading due to track twist.

Using the results of a previous study where lateral wheel/rail force was determined as a function of curvature, the difference in crosslevel between truck centers that will cause Nadal's Limit for wheel climb to be exceeded is determined as a function of curvature. This difference in crosslevel between truck centers is normalized to 31 feet so that maximum track twist, as it is commonly defined, is determined as a function of curvature. It has been found that for less than 6 degrees of curvature and at low speeds, most rail cars can withstand up to 1.5 inches of track twist in 31 feet. The amount of track twist that a car can withstand decreases with increasing curvature, and at 15 degrees of curvature the amount of track twist that most railcars can withstand has dropped to 1.0 inches.
PREFACE

In an effort to increase railway safety, the Office of Research and Development of the Federal Railroad Administration (FRA) is conducting the Track Safety Research Program. In support of this program, the Transportation Systems Center (TSC) has been conducting analytical and experimental studies to determine the relationship between train derailment tendencies and the characteristics of the vehicle and track. TSC is making efforts to determine safety criteria based upon vehicle and track performance. This study has been conducted to determine the effect of track twist on rail vehicle derailment tendencies. Also included in this study is a determination of the effects of the vehicle characteristics on its ability to withstand track twist.

The authors would like to thank Mr. Brandon Schwarz, a student at Northeastern University working at TSC as part of his co-op rotation, for his work plotting graphs, collecting data, and helping to assemble this report.
**METRIC CONVERSION FACTORS**

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*1 in. = 2.54 cm (exactly). For other exact conversions and more detail tables see NBS Misc. Pub. 284. Units of Weight and Measures. Price $2.26 BD Catalog No. C13 10 284.
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EXECUTIVE SUMMARY

This report contains a closed form analysis of track twist as a derailment potential for rail cars. It has been found that unloaded cars with torsionally stiff bodies are more likely to exceed Nadal's Limit for wheel climb on twisted track than are loaded cars with flexible bodies. Of the six typical rail cars that are analyzed in the report, unloaded tank cars are the most likely to exceed Nadal's Limit on twisted track, requiring the least difference in crosslevel between truck centers. When the results of the analysis are normalized to difference in crosslevel in 31 feet, it is found that most rail cars can sustain a track twist of more than 1.5 inches in 31 feet for curves less than 6 degrees. Flat cars are predicted to be the most susceptible to a derailment due to track twist when the results are normalized in this way.

Track twist is the difference in crosslevel between two points on the track. A wheel climb derailment can occur in a curve if the track is twisted. This can happen because track twist causes the vertical load carried by the wheels on one side of the front truck and the vertical load carried by the wheels on the opposite side of the rear truck to decrease while causing the vertical load to increase for the remaining wheels. Lateral forces are necessary to turn the trucks through a curve. If the vertical load, reduced by track twist, is not great enough to support the lateral curving force, the wheel will climb the rail and derail the vehicle.

The characteristics that affect a vehicle's reaction to track twist are determined in this report. The vehicle characteristics that affect the cars reaction to track twist include the carbody weight, the snubber friction, the spring group stiffness, the lateral and longitudinal spacing of the spring/snubber groups, the carbody torsional stiffness, the sidebearing clearance, and the lateral spacing of the side bearings. The characteristics that most influence the cars reaction to track twist are the carbody weight and torsional stiffness and the geometry of the sidebearings. The geometry of the connection between the carbody and truck used in most rail freight vehicles allows over 1.64 inches of difference in crosslevel between centers with only a small amount of wheel unloading. The snubbers can effectively lock the suspension of the vehicle if they exert enough force, which greatly reduces the cars ability to react to track twist. Increasing torsional stiffness and decreasing carbody weight (or load) also decrease the cars ability to comply with track twist.
1. INTRODUCTION

Track twist is the difference in crosslevel between two points along the track. The existence of track twist does not permit the wheels of the rail car to all lie in the same plane. Track twist can occur under several conditions. Track twist occurs at the entry and exit spirals of curves, where the track must twist from the level tangent track up to the superelevation of the curve. Twist can also occur as a defect in the track, caused by environmental conditions or loadings. Joints or places where the ballast does not support the track evenly, such as a road crossing, can become lower than the rest of the track.

When a car travels over track that is twisted, the loads carried by the wheels of the car are redistributed. The vertical loads carried at two diagonally opposite corners of the car will decrease, while the vertical loads will increase at the other two diagonally opposite corners. To support a lateral force acting on a wheel, there must be sufficient vertical force to keep the wheel from climbing over the rail. The analysis done by Nadal (1), which defines a maximum lateral to vertical force ratio (L/V) for a wheel, has been accepted as a criterion for determining if a wheel will climb a rail. On tangent track, unless there are track irregularities, the lateral forces are low, so the possibility of derailment due to track twist is low. Even in smooth curves however, large lateral forces are developed to turn the truck through the curve (2). Track twist can be great enough for a wheel to unload sufficiently so that it can climb the rail and derail the car in a curve.

Track twist is a potential cause of derailments. The Federal Railroad Administration's (FRA) Track Safety Standards limit track twist for the six classes of track in both curves and tangent track (3). The American Railway Engineering Association (AREA), in the Handbook of Railway Engineering, recommends a design maximum of 1 inch twist in 62 feet for exit and entrance spirals to curves (4). Articles have appeared in trade magazines that describe derailments due to track twist, a recent article in Railway Age describes three derailments that were attributed to excessive track twist (5).

Previous analysis of track twist has been limited. An analysis has been done by the Office of Research and Experiments (ORE) of the International Union of Railways for European freight cars (6). The analysis is for two axled vehicles, commonly used in Europe, and is based on empirical measurements of the car's reaction to twist.

The analysis described in this report is a quasi-static analysis to determine the maximum permissible track twist as a function of curvature, based on the vehicle characteristics. The vertical reactions are determined as a function of track twist and
are crossplotted with the results of a previous analysis, which determined lateral wheel/rail force as a function of curvature. Using Nadal's limit the maximum track twist that can be tolerated by a rail car without a wheel climbing the rail is calculated as a function of curvature. This quasi-static analysis is applicable to cars traveling at low speed, when the dynamic response of the car to the track is small.

The vertical reactions are determined as a function of the difference in crosslevel between truck centers by static analysis. If the loads carried by the spring/snubber groups of the truck are known, the vertical reactions to that load by the wheels are determined by using equations of static equilibrium. The loads carried by the four spring/snubbers groups that make up the suspension of the car are determined from three available static equilibrium equations along with one equation based on the carbody torsional flexibility.

The lateral forces acting on the truck as it traverses the curve are obtained from a study done by Blader (7). By using Nadal's Limit, a minimum vertical force necessary to support the lateral force is determined. Once the vertical force is known, the difference in crosslevel between truck centers is determined from the static analysis described in the preceding paragraph. The difference in crosslevel between truck centers is normalized to a difference in crosslevel in 31 feet, which is the usual measure for track twist.
2. ANALYSIS OF VERTICAL WHEEL LOADS

The vertical wheel loads are determined as a function of the difference in crosslevel between truck centers. This is done by modelling the vehicle in two parts. The first part consists of a truck and the loads applied to it through its spring/snubber groups. The vertical wheel loads can be determined as a function of the loads supported by the spring/snubber groups by using equations of static equilibrium. The second part of the model consists of a flexible plate supported at each corner by a spring snubber group. The base of one spring group is out of plane by an amount $Z$ from the plane formed by the base of the other three spring groups. The loads supported by the four spring/snubber groups can be determined from three equations of static equilibrium and one equation based on the carbody torsional equilibrium. The vertical loads carried by the wheels are thus determined as a function of the difference in crosslevel between truck centers.

The vehicle's reaction to track twist is also affected by the geometry of the connection between the carbody and the trucks. An analysis has been done of this geometry to determine its affect on the car's reaction to track twist.

Figure 1 shows sketches of the model used for the truck, the model for carbody, and a sketch of the geometry of the connection between the two.

2.1 THE TRUCKS

The load carried by each wheel of the truck is comprised of the load carried by the spring/snubber groups and the weight of the truck itself. The trucks are assumed to be equalized, that is the loads carried by the two wheels of the truck that are on one rail are the same. The contribution by the spring/snubber groups to the load carried by the wheels can be determined with the use of a free body diagram. Figure 2 shows a free body diagram of the forces acting on the truck at one end of the car, which carries the loads from the spring/snubber groups at corners 2 and 3. Summing moments at corner 3 leads to

$$2L_3(B + H) + 2L_3(B-H) - 2R_2B - W_B = 0$$  \hspace{1cm} (2-1a)

Summing vertical forces leads to

$$2L_2 - 2L_3 = R_2 + R_3 + W_t$$  \hspace{1cm} (2-1b)
FIGURE 1. DIAGRAM OF TRUCK MODEL, CARBODY MODEL, AND CARBODY TO BOLSTER GEOMETRY
These two equations can be solved for the wheel loads in terms of the spring/snubber group loads. The loads carried by the wheels are

\[ L_2 = \frac{1}{4}((R_2 + R_3) + B/H(R_2 - R_3) + W_i) \]  
\[ L_3 = \frac{1}{4}((R_2 + R_3) - B/H(R_2 - R_3) + W_i) \]

A similar analysis on the second truck shows that the loads supported by the wheels of that truck are

\[ L_1 = \frac{1}{4}((R_1 + R_4) - B/H(R_4 - R_1) + W_i) \]  
\[ L_4 = \frac{1}{4}((R_1 + R_4) + B/H(R_4 - R_1) + W_i) \]

2.2 FLEXIBLE PLATE SUPPORTED BY FOUR SPRING/SNUBBER GROUPS

The loads supported by the spring/snubber groups, \( R_1 \) through \( R_4 \), are determined by analyzing the second part of the model, the flexible plate supported at each corner by a spring/snubber group. The analysis follows.
\section*{2.2.1 Carbody Flexibility}

The twist of the carbody when the rail car is subjected to a twist moment is assumed to be proportional to that twist moment. To simplify some of the calculations the torsional stiffness of the carbody has been normalized to the carbody length, i.e.,

\[ \tau = \left( \frac{K_t}{L} \right) \alpha \]  

Where \( \tau \) is the twist moment

- \( K_t \) is the torsional stiffness of the carbody
- \( L \) is the length between truck centers
- \( \alpha \) is the twist through the carbody in radians

\section*{2.2.2 Spring/Snubber Forces}

The reactions at each of the spring/snubber groups can be written in terms of the spring deflections and snubber reactions. The reactions at each corner are written in the form,

\[ R_1 = K\delta_1 + f_1 \]  

\[ R_2 = K\delta_2 + f_2 \]  

\[ R_3 = K(\delta_3 - Z) + f_3 \]  

\[ R_4 = K\delta_4 + f_4 \]

Where \( Z \) is the distance the base of the spring/snubber group at corner 3 of the car is from the plane formed by the bases of the other three spring/snubber groups and \( f \) is the snubber friction force at each corner. \( Z \) is \textit{not} the difference in crosslevel between truck centers, because the spring groups are outside the contact points between the wheels and the rails. The two values are related later in this report.

The deflections are written in terms of the downward displacement of the center of the carbody, \( \delta_0 \), plus the rotations \( \theta \) and \( \phi \) about the geometric center plus the displacements due to the twist of the carbody caused by the torque applied, \( \Delta_1 \) and \( \Delta_2 \). The carbody is assumed to deflect in the manner shown in Figure 3. The deflections at each of the corners are then

\[ \delta_1 = \delta_0 - A\theta - B\phi + \Delta_1 \]  

\[ \delta_2 = \delta_0 + A\theta - B\phi - \Delta_2 \]  

\[ \delta_3 = \delta_0 + A\theta - B\phi - \Delta_2 \]  

\[ \delta_4 = \delta_0 - A\theta - B\phi - \Delta_1 \]
FIGURE 3. ASSUMED DEFLECTION OF CAR BODY
Three independent equations are obtained from equilibrium of the forces and moment equilibrium about $\theta$ and $\phi$,

\[
\begin{align*}
\sum F_2 &= R_1 + R_2 + R_3 + R_4 - W = 0 \quad (2-6) \\
\sum M_\theta &= (R_1 + R_4)A - (R_2 + R_3)A - Wa = 0 \quad (2-7) \\
M_\phi &= (R_1 + R_2)B - (R_3 + R_4)B - Wb = 0 \quad (2-8)
\end{align*}
\]

Two more equations are obtained by relating the deflections of the corners of the idealized carbody, $\Delta_1$ and $\Delta_2$, to the torsional flexibility of the carbody.

\[
\begin{align*}
\Delta_1 &= B_{a1} \quad (2-9) \\
\Delta_2 &= B_{a2} \quad (2-10)
\end{align*}
\]

The twist angles $\alpha_1$ and $\alpha_2$ are written in the form

\[
\begin{align*}
\alpha_1 &= \frac{(R_4 - R_1)AB + W_{ab}}{K_c} \quad (2-11) \\
\alpha_2 &= \frac{(R_4 - R_1)AB + W_{ab}}{K_c} \quad (2-12)
\end{align*}
\]

Equations (2-6-10) are rewritten in terms of the five unknowns $\delta_0$, $\theta$, $\phi$, $\Delta_1$, and $\Delta_2$, and solved. These five unknowns are determined as a function of $Z$.

\[
\begin{align*}
\delta_0 &= \frac{W/4K + Z/4}{1 + \xi} \quad (2-13) \\
\theta &= \frac{Z/4A - Wa/4KA^2}{1 + \xi} \quad (2-14) \\
\phi &= \frac{Z/4B - (W/4KB)(1 + \xi)b/B + ab/AB)}{1 + \xi} \quad (2-15) \\
\Delta_1 &= \frac{Z/4 - W/(4KB)(1 + \xi)((b/B - ab/AB) - (f_1 - f_2 + f_3 - f_4)/K)}{[1 + 1/\xi]} \quad (2-16) \\
\Delta_2 &= \frac{Z/4 - W/(4k)(a/b/AB - (1 + \xi)(b/B) - (f_1 - f_2 + f_3 - f_4)/K)}{[1 + 1/\xi]} \quad (2-17)
\end{align*}
\]

where $\xi$ is the dimensionless group

\[
\xi = 2KAB^2 K. \quad (2-18)
\]
The normal reactions at each spring/snubber group follow from substitution into equations (2-4a-d),

\[ R_1 = \frac{W}{4} \left( 1 + \frac{a}{A} + \frac{b}{B} + \frac{\xi_{ab}}{AB(1 + \xi)} \right) - \frac{(KZ - (f_1 - f_2 + f_3 - f_4))}{4(1 + \xi)} \]  

(2-19a)

\[ R_2 = \frac{W}{4} \left( 1 - \frac{a}{A} + \frac{b}{B} - \frac{\xi_{ab}}{AB(1 + \xi)} \right) + \frac{(KZ - (f_1 - f_2 + f_3 - f_4))}{4(1 + \xi)} \]  

(2-19b)

\[ R_3 = \frac{W}{4} \left( 1 - \frac{a}{A} - \frac{b}{B} + \frac{\xi_{ab}}{AB(1 + \xi)} \right) - \frac{(KZ - (f_1 - f_2 + f_3 - f_4))}{4(1 + \xi)} \]  

(2-19c)

\[ R_4 = \frac{W}{4} \left( 1 + \frac{a}{A} - \frac{b}{B} - \frac{\xi_{ab}}{AB(1 + \xi)} \right) + \frac{(KZ - (f_1 - f_2 + f_3 - f_4))}{4(1 + \xi)} \]  

(2-19d)

2.3 TRUCK-CARBODY CONNECTION GEOMETRY

Due to the geometry of the connection between the carbody and the trucks, the car can react to an amount of track twist with only a small change in the loads supported by the wheels. This occurs when the carbody goes from resting solely on the centerplates of the trucks to resting on the centerplates and sidebearings. The carbody can also separate completely from either centerplate and this is also due to the geometry of the connection between the carbody and the centerplate.

2.3.1 Rotation of Carbody about Edge of Centerplate

The carbody will begin to rotate about the edges of the centerplates toward side bearings when the difference in the loads carried on each side of a truck becomes great enough. The centerplate edges that the carbody rotates about will be diagonally opposite each other. The change in load carried by the spring/snubber group necessary for the carbody to begin to rotate about the edge of the centerplate is determined with the use of the free body diagram shown in Figure 4. The load carried by the spring/snubber groups are \( R_A \) and \( R_B \), and the percentage reduction in the load \( R_A \) is \( P_r \).

The carbody will just begin to rotate about the edge of the centerplate when load due to the carbody weight is carried at the edge of the centerplate. Summing moments about side b of the bolster, where the spring/snubber force \( R_B \) acts, leads to

\[ R_A = \frac{(B - r_{cp})/B}{W/4} \]  

(2-20)
The percentage decrease in the load $R_a$ carried by the spring/snubber group from the nominal load of $W/4$ is then

$$P_r = \frac{r_{cp}/B}{W/2} \times 100$$

(2-21)

This equation shows that the reduction in load carried by the spring/snubber group necessary for the carbody to rotate about the centerplate depends upon only the geometry, not upon the weight of the vehicle. For a typical freight car truck equipped with a 14 inch diameter centerplate and a 77 inch lateral separation between spring groups, this reduction in load carried by the spring/snubber group (not the load carried by the wheels) is 18%.

![Free Body Diagram of Bolster with Impending Bolster Rotation About Edge of Centerplate](image)

**FIGURE 4.** FREE BODY DIAGRAM OF BOLSTER WITH IMPENDING BOLSTER ROTATION ABOUT EDGE OF CENTERPLATE

### 2.3.2 Effect of Sidebearing Clearance

Once the carbody begins to rotate about the edge of the centerplate, there is very little resistance to the rotation of the carbody relative to the bolsters, until the carbody contacts the sidbearings. If the rotational resistance between the carbody and the corner of the centerplate is neglected, the amount of 'free twist' due to the sidebearing clearance can be determined from the geometry of the rail car. There is very little change in the load carried by the wheels while the car is rotating from resting completely on the centerplates to resting on both the edge of the centerplates and on the sidebearings because the load is applied through the edge of the centerplate during the rotation.
The 'free twist' can be thought of as a two step process. During the first step of this process, as the car travels down track that increasingly twists, the forward bolster rotates to the side, about corner 2 of the car in the $\phi$ direction while the carbody rotates forward in the $\theta$ direction until the carbody comes in contact with the sidebearing at corner 2 of the car. The carbody does not rotate to the side in the $\phi$ direction. Figure 5 shows the front and rear bolsters and carbody in their original positions, and also rotated to their new positions. After the carbody and bolster have rotated in such a manner, the forward bolster has rotated by an angle equal to $\Delta_{sb}/(r_{sb}-r_{cp})$, while the rear bolster has remained in its original position. The side bearing clearance has been taken up on the forward bolster so that the carbody is supported by the edge of the centerplate and the side bearing at corner 2. The carbody is still in full contact with the centerplate on the rear bolster. During the second step, as track twist increases, the forward bolster continues to rotate to the side about corner 2. The carbody rotates with the bolster because the forward sidebearing clearance was closed at the completion of step 1. The carbody also continues to rotate forward as in step 1. Step 2 reaches completion when the carbody contacts the side bearing at corner 4, on the rear bolster. The forward bolster has rotated an additional $\Delta_{sb}/(r_{sb}-r_{cp})$ while the rear bolster remains stationary. Figure 5 shows the bolsters and the carbody after the second step. The total angular displacement of the forward bolster is the sum of step 1 and step 2, $2\Delta_{sb}/(r_{sb}-r_{cp})$. The amount of track twist that the sidebearing clearances allow is then

$$T_s = 4H\Delta_{sb}/(r_{sb}-r_{cp})$$

(2-22)

This total amount of 'free twists' does not depend upon what order the body comes into contact with the sidebearings. When the rail car is twisted such that it is in a 'free twist' state, it is difficult to know exactly where the carbody is in relation to the bolster; it may be in full contact with a single centerplate or it may be in contact with the edge of a centerplate and an adjacent sidebearing. The exact position of the carbody when it is in a 'free twist' state has little effect upon the loads carried by the wheels.

The amount of free twist, $T_s$, is independent of any amount of initial twist. The car is able to accept an additional amount of 'free twist' without any change in load carried by the spring/snubber groups. The results of the geometric analysis can be superposed on the results of the static analysis. The total twist is the sum of the initial twist and the 'free twist'.

-11-
FIGURE 5. SIDEBEARING CLEARANCE EFFECT ON CAR'S REACTION TO TRACK TWIST.
2.3.3 Maximum Wheel Unloading

The carbody begins to separate from the centerplate when the load the bolster supports is carried solely by a side bearing. Figure 6 is a free body diagram of the bolster showing the total load being carried by a side bearing. Summing moments about side b of the bolster, where the spring/snubber force $R_b$ acts, leads to

$$R_a = \frac{(B - r_{sb})/B}{W/4}$$

The percentage reduction in load $R_a$ from $W/4$ is then

$$Pr = \frac{r_{sb} / B}{W} \times 100.$$  

Again this reduction in load depends only upon the geometry. For a typical rail car with a 50 inch side bearing lateral separation and a 77 inch lateral separation between spring groups, this reduction in load carried by the spring/snubber group is 65%.

![Free Body Diagram of Bolster with Impending Carbody/Centerplate Separation](image)

**FIGURE 6.** FREE BODY DIAGRAM OF BOLSTER WITH IMPENDING CARBODY/CENTERPLATE SEPARATION

Once the centerplate no longer shares the load and the load is completely carried by the side bearing, the reactions at each spring/snubber group no longer depend upon the difference in crosslevel between truck centers. This is because the carbody is not capable of transmitting a torque to the bolster through just the side bearing. The sidebearing can only support a vertical load, it cannot support a torque by itself. Because of this, it is not possible to completely unload a wheel unless there is a lateral force acting on the axle.
2.4 GENERAL BEHAVIOR OF MODEL

Now that all the components of the model have been analyzed, the car's reaction to track twist can be described. The difference in crosslevel between truck centers, \( T \), is related to \( Z \), the distance the base of the spring/snubber group at corner 3 is out from the plane formed by the bases of the other spring/snubber groups, by geometry. Since the spring/snubber groups are separated by a distance \( 2B \) and the contact points between the wheels and the rails are separated by a distance \( 2H \), then

\[
Z = B(T/H)
\]  
(2-25)

The load carried by the wheels are related to the loads carried by the spring/snubber groups by equations (2-2a-d). The final equations for the loads carried by the spring/snubber groups can be substituted into equations (2-19a-d) and the preceding expression for \( Z \) in terms of \( T \) can be substituted to arrive at an expression for the wheel load as a function of the difference in crosslevel between the truck centers. The equations are

\[
L_1 = 1/8(W1 + a) + \frac{b}{A} + \frac{\xi_{ab}}{AB} \left[ 1 + \xi \right] \left( \frac{KB(T/H)}{H} \right) - 2W_f
\]  
(2-26a)

\[
L_2 = 1/8(W1 - a) + \frac{b}{A} - \frac{\xi_{ab}}{AB} \left[ 1 + \xi \right] \left( \frac{KB(T/H)}{H} \right) - 2W_f
\]  
(2-26b)

\[
L_3 = 1/8(W1 - a) - \frac{b}{A} + \frac{\xi_{ab}}{AB} \left[ 1 + \xi \right] \left( \frac{KB(T/H)}{H} \right) - 2W_f
\]  
(2-26c)

\[
L_4 = 1/8(W1 + a) - \frac{b}{A} - \frac{\xi_{ab}}{AB} \left[ 1 + \xi \right] \left( \frac{KB(T/H)}{H} \right) - 2W_f
\]  
(2-26d)

The wheel loads at points of transition from resting on the centerplate, to rotating about the edge of the centerplate and onto the sidebearings, and separation from the centerplate depend upon the vehicle weight as well as the geometry, and so it is more convenient to determine these points using the loads on the spring/snubber groups rather than the wheel loads.
Figure 7 shows a plot of wheel load vs. difference in crosslevel between truck centers. In the first section of the graph, the car is resting on both centerplates and up until the loads carried by the spring/snubber groups has changed by 18%, the twist is reacted by only the suspension and the torsional flexibility of the car body. Once the load carried by the spring/snubber group has changed sufficiently, which occurs at point A, the car 'free twists', with no change in the wheel load until the carbody has come into contact with two sidebearings, which occurs at point B. While the carbody is in contact with two sidebearings and both centerplates, the track twist is again reacted by the suspension and the torsional flexibility of the carbody. Once the spring/snubber group has unloaded sufficiently, at 65% unloading, the carbody will begin to separate from the centerplate, and the wheel loads will not change when the track twist is increased. The snubber friction has been taken to be zero to simplify the description of the plot. The affect of snubber friction on the vehicles reaction to track twist is discussed in detail in the following section.
A - LOAD AT EDGE OF CENTERPLATE
B - LOAD SHARED BY CENTERPLATE AND SIDE BEARING
C - ALL OF LOAD CARRIED BY SIDE BEARING

FIGURE 7. WHEEL LOAD VS. DIFFERENCE IN CROSSLEVEL BETWEEN TRUCK CENTERS
3. SNUBBER FRICTION: WORST SNUBBER FRICTION SCENARIO

For almost any situation that the car could be in, each of the snubber forces could be anything from \( +f \), helping to support the car with the maximum available friction force, to \( -f \), trying to keep the springs compressed with the maximum available friction force, the exact value being dependent upon the recent history of the car. Several different scenarios, involving extremes of the snubber forces, of the recent history of the car can be envisioned. In the appendix, six different scenarios, which bound the possible extremes of the snubber friction, are analyzed. The analysis of the scenario that causes the greatest amount of wheel unloading for a given amount of track twist follows.

In this scenario, which is the last scenario analyzed in the appendix, the car starts on track that is twisted by the maximum amount, the track then evenly twists in the opposite direction. In this way, the car goes from the maximum twist in one direction through to the maximum twist in the opposite direction.

Initially, when the car is on the track that is twisted to one extreme, the spring/snubber group at corner 3 is overloaded by 65%. The weight of the carbody is carried by two sidebearings. The amount of initial displacement of the base of the spring/snubber group at corner 3, \( Z^* \), necessary for this to occur can be determined from

\[
1.65\frac{W}{4} = \frac{KZ^*/4 - f}{1 + \xi} \quad (3-1a)
\]

\[
Z^* = \frac{0.65\left(1 + \xi\right)W - 4f}{K} \quad (3-1b)
\]

Since the snubber forces must reverse direction, the track twist is reacted only by the carbody. Until the load carried by the spring/snubber groups have changed by 2\( f \), the load carried at corner 3 is given by

\[
R_3 = \frac{W/4 - KZ^*/4}{1 + \xi} - \frac{f - KZ^*/4}{8AB^2} \quad (3-2)
\]

At 18\% overload of \( R_3 \), the carbody begins to rotate back onto the centerplates. This may or may not occur before the snubber break out, depending upon the weight of the carbody. The carbody will begin to rotate towards the opposite sidebearings when \( R_3 \) has decreased from \( W/4 \) by 18\%. After the snubbers have broken out, the load carried by the spring/snubber group at corner 3 is given by

\[
R_3 = \frac{W}{4 - \frac{KZ - Z^*/4 - f}{1 - \xi}} \quad (3-3)
\]
Again, the carbody will separate from the centerplates when \( R_3 \) has decreased by 65% from \( W/4 \).

Using this notation the initial displacement of the base of the spring/snubber group at corner 3, \( Z \), is zero, even though the track is initially twisted. The expression \((Z+Z^*)\) is zero when the car is on level track. Figure 8 shows a graph of wheel loading vs. difference in crosslevel between truck centers for an unloaded 100 ton covered hopper car, with \((Z+Z^*)\) is taken to be the difference in crosslevel between truck centers.

![Graph showing wheel load vs. difference in crosslevel between truck centers for a 100 ton covered hopper car.](image)

**FIGURE 8. WHEEL LOAD VS. DIFFERENCE IN CROSSLEVEL BETWEEN TRUCK CENTERS FOR A 100 TON COVERED HOPPER CAR**
4. WHEEL UNLOADING OF TYPICAL FREIGHT CARS

A set of curves for 6 common types of rail cars of percent wheel unloading vs. difference in crosslevel between truck centers is shown in Figure 9. The snubber friction is assumed to be the 'worst case', the scenario that was analyzed in the preceding section. The characteristics used for each of these cars are the characteristics for the largest subgroup of each car type in reference [8]. Table 1 lists the characteristics used in the model for each of the cars. From this graph it can be seen that the cars that are the most susceptible to wheel unloading due to track twist are the tank car and the flat car with bulkhead.

FIGURE 9. PERCENT WHEEL UNLOADING VS. DIFFERENCE IN CROSSLEVEL BETWEEN TRUCK CENTERS FOR SIX COMMON TYPES OF RAIL CARS
### TABLE 1. PHYSICAL CHARACTERISTICS OF SIX COMMON RAIL CARS

<table>
<thead>
<tr>
<th>Car Type</th>
<th>Torsional Stiffness (ln^2 kips/rad)</th>
<th>Carbody Weight (kips)</th>
<th>Truck Center Length (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covered Hopper</td>
<td>2.34x10^8</td>
<td>42.3</td>
<td>492</td>
</tr>
<tr>
<td>Open Hopper</td>
<td>4.22x10^6</td>
<td>31.5</td>
<td>384</td>
</tr>
<tr>
<td>Flat with Bulkhead</td>
<td>2.19x10^8</td>
<td>36.2</td>
<td>480</td>
</tr>
<tr>
<td>Flat</td>
<td>3.50x10^7</td>
<td>50.4</td>
<td>798</td>
</tr>
<tr>
<td>Box</td>
<td>2.01x10^7</td>
<td>47.2</td>
<td>490</td>
</tr>
<tr>
<td>Tank</td>
<td>1.99x10^9</td>
<td>32.0</td>
<td>312</td>
</tr>
</tbody>
</table>

**100 Ton Truck Average Data**

- Total Weight ($W_t$): 9.27 kips
- Spring Group Stiffness ($K$): 22 kips/inch
- Snubber Force ($f$): 4 kips
- Distance Between Side Bearings ($2r_{sb}$): 50 inches
- Side Bearing Clearance ($\Delta_{sb}$): 0.25 inches
- Centerplate Diameter ($2r_{cp}$): 14 inches
- Lateral Spring Group Spacing ($2B$): 77 inches
- Lateral Wheel/Rail Contact Patch Separation ($2H$): 59 inches

Data for largest sub-population of vehicle type, taken from *Engineering Data Characterizing the Fleet of U.S. Railway Rolling Stock*, Volumes I and II, FRA/ORD-81/75.1, November 1981, F. DiMasi.
5. EFFECT OF VEHICLE CHARACTERISTICS

As illustrated in Figure 9, different rail cars have different reactions to track twist. This is due to the differences in physical characteristics of the cars. The characteristics of the rail car that affect its reaction to track twist are the location of the center of gravity, the vehicle weight, the snuber friction, the spring group stiffness, the lateral and longitudinal spacing of the spring/snubber groups, the carbody torsional stiffness, the side bearing clearance, and the lateral spacing of the side bearings.

The two characteristics that have the greatest affect on the cars reaction to track twist are the carbody weight, W, and the torsional stiffness of the carbody, Kc. In general, the heavier the car, the better able it is to withstand track twist. This can be seen by inspection of equations (2-18a - d).

The amount of twist a rail car can tolerate is inversely related to the stiffness of the carbody. Torsional stiffness is one measure of the coupling that exists between the two trucks. Decreasing the stiffness decreases the coupling between the two trucks. The two extremes are when the torsional stiffness approaches infinity and when it approaches zero. As the torsional stiffness approaches infinity, the carbody acts as a rigid plane. Applying the limit to equation (c)

\[ \lim_{K_c \to \infty} R_3 = \frac{W}{4[(1-a/A)(1-b/B)]} \]  

As the torsional stiffness of the car body approaches zero, the trucks are effectively uncoupled and the normal reactions at each spring/snubber group become independent of track twist. The reaction R3 at spring/snubber group (3) becomes

\[ \lim_{K_c \to 0} R_3 = \frac{W}{4[(1-a/A)(1-b/B)(KZ/4)]} \]  

This equation is independent of Z, which is the distance the base of the spring/snubber group at corner 3 of the car is from the plane formed by bases of the three other spring/snubber groups. Consequently the loads carried by each of the wheels, L1, L2, L3, L4 are independent of T, the difference in crosslevel between truck centers, which is proportional to Z. Figure 10 is a plot of the difference in crosslevel between truck center for the unloading of the third spring/snubber group (R3=0) vs. torsional stiffness. For Kc greater than approximately 10x10^7 Kip-inch^2/radian the carbody acts like a rigid plane.
Effect of torsional stiffness on an unloaded 100 ton covered hopper car.

FIGURE 10. DIFFERENCE IN CROSSLEVEL BETWEEN TRUCK CENTERS FOR WHEEL UNLOADING VS. CARBODY TORSIONAL STIFFNESS
Many of the other vehicle characteristics that are related to the car's ability to conform to track twist are within a limited range of values, whether due to regulation or manufacturing standardization. These characteristics include: the centerplate radius, which has the range of 12 inches for a 70 ton truck to 16 inches for a 125 ton truck; the spring nest spacing on a truck, which has a range of 77 inches for a 70 ton truck to 79 inches for a 125 ton truck; the side bearing spacing which is typically 0.25 inch ± 0.125 inch; the spring nest stiffness ranges from 22 kips/inch for a 70 ton truck to 27 kips/inch for a 125 ton truck.

The snubber friction is also usually within a limited range of values, but often for new cars the snubbers tend to stick, and can exert a large force. The effect of increasing snubber friction is to decrease the ability of the rail car to comply with track twist. The snubber friction may range from zero up to the magnitude of the load applied (if the snubber is seized). The effect of the snubber friction in equations (2-19a - d) at first may appear counter intuitive in that the reactions appear to reach zero at even the slightest amount of difference in crosslevel between truck centers for a very high snubber friction. However, for this to occur the snubber force would have to be greater than the load applied to the snubber. The effect of the suspension stiffness approaching infinity is equivalent to the snubbers seizing. Taking the limit of equation (2-19c), the reaction force is of the form

$$\lim_{K \to \infty} R_3 = \frac{W}{4\left[(1 - a/A)(1 - b/B)\right]} - \left(\frac{ZKc}{8AB^2}\right)$$

(5-3)

The snubber force cannot exceed this value, and since this equation is less than equation (2-19c), the rail car can comply with some track twist, due to the flexibility of the carbody (and the sidebearing clearance) even if the snubbers are seized. For a car with a very flexible body, such as a typical gondola car, the snubbers seizing may not affect the car's reaction to track twist significantly, while for a car with a very stiff body, such as a typical tank car, the car's reaction to track twist will be affected and the car will be more susceptible to wheel climb in a curve.
6. CONCLUSION AND SUMMARY OF VERTICAL WHEEL LOAD ANALYSIS

This completes the analysis of the change in vertical forces due to track twist. The analysis is a static analysis to determine the vertical loads carried by each of the wheels. The analysis does ignore any lateral forces acting on the trucks. Even though the lateral forces are neglected, the results of the analysis are valid because the net lateral forces acting on the truck are much less than the net vertical forces acting on the truck. The lateral forces acting on the carbody are adequately taken into account by moving the center of gravity of the carbody inward or outward from the center of the curve by the appropriate amount.

Because it is difficult to know the force that is being exerted by each of the snubbers, it is consequently difficult to know the loads carried by each of the wheels. A number of scenarios of the recent history of the car which produce several extreme conditions of the snubber forces have been analyzed. A worst case for the snubber friction, which produces the most wheel unloading of the six cases analyzed, has been determined.

The analysis shows that the carbody weight and torsional stiffness as well as the sidebearing clearance are the primary vehicle characteristics that influence the change in the loads carried by the wheels due to track twist. The more torsionally stiff that a carbody is, the greater the amount of wheel unloading for a fixed amount of track twist. Increasing vehicle weight however, decreases the amount of wheel unloading for a given amount of track twist. The sidebearing clearance has the effect of allowing the vehicle to react to a range of track twist without changing the vertical loads carried by the wheels. Increasing the sidebearing clearance, increases this amount of 'free twist'.

Due to the geometry of the bolster, where the carbody and truck join, a wheel cannot be completely unloaded due to track twist. A lateral force acting on the truck is necessary to completely unload a wheel. If the track twist is large enough, the carbody will be supported by only one sidebearing on a truck. If the carbody lifts from the centerplate, then there is nothing restraining any relative lateral and longitudinal movement between the carbody and the truck. The sidebearing provides only vertical support.
7. CURVING MECHANICS

The curving behaviour of rail vehicles has been extensively studied analytically (2,7). These studies can be broken down into two general categories, steady state curving studies, where the track is assumed to be free of irregularities and a quasi-static analysis is done, and dynamic curving, where the dynamic response of the vehicle to track irregularities is determined. These studies, both steady state and dynamic, have shown that the largest lateral wheel/rail force occurs at the lead outer wheel of the truck. The lateral forces developed by a truck in a curve have been shown to be primarily a function of wheel load for a given degree of curvature in a study of steady state curving done by Weinstock and Greif(2).

An extensive study of the curving behaviour of a 100 ton covered hopper car has recently been done by Blader (7). The curving behaviour was analyzed for various track irregularities, as well as for steady state curving. A dynamic model of a freight car was used for this study, both for the dynamic and steady curving analysis. For the steady curving analysis, the rail was assumed to be free of irregularities. The computer program SIMCAR, used in this study, contains a 16 degree of freedom dynamic model. The model represents the car suspension in a piecewise linear fashion. The computer program can predict the dynamic response of a freight vehicle to various track irregularities and can also predict the vehicle steady state response to a smooth curve.

In the study, the lateral force acting on the lead outer wheel was determined as a function of curvature, at balance speed, for a loaded 100 ton hopper car, and these results are shown in Figure 11. Since the lateral curving forces acting on a truck have been shown to be primarily a function of the normal wheel load the lateral force that would act on the lead outer wheel of a rail car with a different weight can be estimated from the results given by Blader for 100 ton hopper car. Since the nominal wheel load is constant, at 33 kips, the lateral force scale on the ordinate can be replaced by an $L/V$ scale by dividing the lateral force by the nominal wheel load.
FIGURE 11. LATERAL WHEEL/RAIL FORCE VS. CURVATURE FOR A LOADED 100 TON HOPPER CAR
8. TRACK TWIST AND CURVATURE: FACTORS INFLUENCING VEHICLE DERAILMENT TENDENCIES DUE TO TRACK TWIST

Nadal's Limit defines a lateral to vertical force ratio that will cause a wheel to climb over the rail head. Figure 12 shows a free body diagram of a wheel with the forces acting on it. From equilibrium of the contact point between the wheel and rail the lateral and vertical forces are found to be

\[ L = N \sin \delta - \mu N \cos \delta \]  
\[ V = N \cos \delta - \mu N \sin \delta \]

The ratio of these two forces, which is Nadal's Limit, is

\[ NL = \frac{\tan \delta - \mu}{1 + \tan \delta} \]  

By comparing Nadal's Limit to the L/V ratios from Figure 11, then a maximum permissible percent wheel unloading can be determined. The percent wheel unloading is given by

\[ \beta = 1 - \frac{L/V}{NL} \]

where \( \beta \) is the percent wheel unloading

NL is Nadal's Limit
L/V is the nominal L/V ratio

\[ L = N \sin \delta - \mu_g N \cos \delta \]
\[ V = N \cos \delta + \mu_g N \sin \delta \]

\[ \frac{L}{V} = \frac{\tan \delta - \mu_g}{1 + \mu_g \tan \delta} \]

FIGURE 12. NADAL'S LIMIT FOR IMPENDING MOTION OF FLANGING WHEEL
Table 2 lists the wheel unloading necessary to cause wheel climb at different degrees of curvature for three different coefficients of friction, 0.375, 0.500, 0.625. By using this table and the equations derived for wheel unloading as a function of difference in crosslevel between truck centers (2-26a - d), the maximum difference in crosslevel between truck centers as a function of curvature can be determined.

### Table 2. Wheel Unloading and Curvature

<table>
<thead>
<tr>
<th>Curvature (Degrees)</th>
<th>L/V</th>
<th>Wheel Unloading (coefficient of friction = 0.625)</th>
<th>0.500</th>
<th>0.375</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.0</td>
<td>0.60</td>
<td>0.14</td>
<td>0.29</td>
<td>0.43</td>
</tr>
<tr>
<td>12.5</td>
<td>0.53</td>
<td>0.24</td>
<td>0.37</td>
<td>0.49</td>
</tr>
<tr>
<td>10.5</td>
<td>0.47</td>
<td>0.33</td>
<td>0.45</td>
<td>0.55</td>
</tr>
<tr>
<td>7.5</td>
<td>0.40</td>
<td>0.43</td>
<td>0.53</td>
<td>0.62</td>
</tr>
<tr>
<td>5.0</td>
<td>0.27</td>
<td>0.61</td>
<td>0.68</td>
<td>0.74</td>
</tr>
<tr>
<td>2.5</td>
<td>0.07</td>
<td>0.90</td>
<td>0.92</td>
<td>0.93</td>
</tr>
<tr>
<td>1.5</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

For a wheel with a flange angle of 67 degrees, and a coefficient of friction of

- NL = 0.700 for 0.625
- NL = 0.850 for 0.500
- NL = 1.050 for 0.375

Figure 13 shows a plot of difference in crosslevel between truck centers vs. curvature for a 100 ton unloaded covered hopper car. There are several characteristics of the curve that should be noted. There are two segments to the curve. In the segment from tangent track (0 degrees of curvature on the graph,) to 6.5 degrees of curvature the difference in crosslevel is constant. This happens because there is a maximum amount of wheel unloading that can occur due to track twist, and for low degrees of curvature this is not enough vertical wheel unloading to cause a wheel to climb the rail. However, the carbody begins to separate from the centerplate of one of the trucks. When the carbody lifts from the centerplate there is nothing restricting any lateral or longitudinal movement between the carbody and the truck. For the section of the curve between 6.5 degrees and 15.0 degrees, track twist alone can cause sufficient wheel unloading to cause Nadal's Limit for wheel climb to be exceeded. For this graph, the worst case has been assumed for the snubber frictions. (See section 3) For all curvature shown on this graph, the 'free twist' due to the sidebearings has been taken up. In other words
sufficient unloading to allow a wheel to climb the rail for curves less than 6.5 degrees does not occur until the carbody is in contact with a sidebearing on each truck. This means that at minimum a car with a 0.25 inch-sidebearing clearance can withstand a difference in crosslevel between truck centers of 1.64 inches before the wheel unloading is great enough to allow a wheel to climb.

**Figure 13.** Difference in Crosslevel Between Truck Centers vs. Curvature, Unloaded 100 Ton Hopper Car
Figure 14 shows a plot of difference in crosslevel between truck centers for the six cars that were analyzed for wheel unloading as a function of difference in crosslevel between truck centers (see Figure 9). The characteristics of the plot are nearly the same as for the previous figure discussed in the preceding paragraph. The snubber friction again was the worst case. Each of these cars (with 0.25 inch sidebearing clearance) can withstand at least the 'free twist', that is 1.64 inches difference in crosslevel between truck centers, before sufficient unloading occurs to allow a wheel climb derailment.

Figure 15 shows the results presented in Figure 14 normalized from difference in crosslevel between truck centers to track twist in 31 feet. This has been done by dividing the difference in crosslevel between truck centers by the truck center spacing and multiplying this by 31 feet. The implicit assumption in doing this is that the track twist is constant. This curve has similar characteristics to the previous two figures, where for less than 5 degrees of curvature track twist does not cause enough wheel unloading to allow a wheel climb derailment, but track twist can cause sufficient unloading at higher degrees of curvature. It is possible for track twist to cause the carbody and a truck to separate at any curvature and on tangent track.

There are two factors, outside of the specific vehicle characteristics, that influence the vehicles reaction to track twist, and these are the coefficient of friction between the wheel and rail and the speed that the vehicle is moving through the curve.

The wheel/rail friction coefficient affects the value of Nadal's Limit. The lower the coefficient of friction, the higher Nadal's Limit becomes. This means that less vertical force is required to support a given lateral force, which in turn means that a vehicle can withstand a greater amount of track twist if the wheel/rail friction coefficient is lowered. Figure 16 shows a plot of maximum track twist vs. curvature for a 100 ton unloaded covered hopper car. There are three lines on the graph, for three different wheel/rail friction coefficients. The figure shows that the wheel/rail friction coefficient has a greater influence at high degrees of curvature. This occurs because the maximum unloading does not depend upon the wheel/rail coefficient of friction.

Unfortunately, the wheel/rail friction coefficient is difficult to know precisely. Friction coefficients between the wheel and rail higher than 0.5 have been observed during field tests (9). The friction between the wheel and rail is influenced by environmental factors, such as the amount of use that the track gets and the weather. Rain greatly reduces the amount of available friction between the wheel and rail.
The speed that the car travels through the curve affects the vertical force supported by the outer wheels. Increasing the speed that the vehicle passes through the curve, increases the load supported by the outer wheels. This increase in load can offset the unloading caused by track twist. Because the greatest lateral force acts on the lead outer wheel of the truck, if the car is traveling above balance speed, it can withstand a greater amount of track twist than if it is traveling below balance speed.

When the car is traveling through a curve above balance speed, centrifugal force pushes the center of gravity outward. (The model used to determine the amount of wheel unloading due to track twist accommodates a center of gravity that does not coincide with the geometric center of the car.) The distance from the centerline of the track that the center of gravity of the car is pushed out by centrifugal force can be determined if the speed of the vehicle and the superelevation of the track are known. The lateral displacement of the center of gravity is due to two factors, the displacement due to the compliance of the suspension and the displacement due to the tilting of the car. The displacement due to the suspension can be determined from a free body diagram and the displacement due to the tilting of the car can be determined from the geometry of the situation. A free body diagram and an illustration of the geometry is shown in Figure 17. By solving the equations of static equilibrium, the lateral excursion of the center of gravity can be determined.

Figure 18 shows maximum track twist vs. curvature for a 100 ton unloaded covered hopper car with three different curves, one for three inches under balance speed, one for balance speed, and the third for three inches over balance speed. The figure shows that superelevation uniformly shifts the curve, and that the effect is independent of curvature. It also illustrates clearly that operating above balance speed increases the amount of track twist that a vehicle can withstand in a curve.
FIGURE 14. DIFFERENCE IN CROSSLEVEL BETWEEN TRUCK CENTERS VS. CURVATURE, SIX DIFFERENT RAIL CARS
FIGURE 15. TRACK TWIST IN 31 FEET VS. CURVATURE, SIX DIFFERENT RAIL CARS

LEGEND

<table>
<thead>
<tr>
<th>Car Type</th>
<th>Truck Centers</th>
<th>Weight (Kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covered Hopper Car</td>
<td>41</td>
<td>42.3</td>
</tr>
<tr>
<td></td>
<td>41</td>
<td>31.5</td>
</tr>
<tr>
<td>Open Hopper Car</td>
<td>32</td>
<td>31.5</td>
</tr>
<tr>
<td>Flat Car w/ bulkhead</td>
<td>40</td>
<td>36.2</td>
</tr>
<tr>
<td>Flat Car w/o Bulkhead</td>
<td>67</td>
<td>50.4</td>
</tr>
<tr>
<td>Box Car</td>
<td>41</td>
<td>47.2</td>
</tr>
<tr>
<td>Tank Car</td>
<td>26</td>
<td>32.0</td>
</tr>
</tbody>
</table>

TABLE 3. LOAD FACTORS FOR ADJACENT TRACKS
FIGURE 16. MAXIMUM PERMISSABLE TRACK TWIST IN 31 FEET VS. CURVATURE, UNLOADED 100 TON HOPPER CAR, LINES OF CONSTANT WHEEL/RAIL COEFFICIENT OF FRICTION
\[ F_L = W(\Delta z/2l - V^2/gR) \]
\[ \sum F_x = 0 : -F_L - 2K_x X = 0 \]
\[ \sum F_y = 0 : -W - K_y Y_a - K_y Y_b = 0 \]
\[ \sum M_0 = 0 : F_L A - (B - A/2B (Y_b - Y_a) + X) - 2K_y Y_b B = 0 \]

Lateral Displacement of C.G.

\[ X_{cg} = \left[ A/2B(Y_b - Y_a) + X \right] + X_1 \]

**FIGURE 17. FREE BODY DIAGRAM FOR DETERMINING LATERAL EXCURSION OF CARBODY CENTER OF GRAVITY IN A CURVE**
FIGURE 18. MAXIMUM PERMISSIBLE TRACK TWIST IN 31 FEET VS. CURVATURE,
UNLOADED 100 TON HOPPER CAR, LINES OF CONSTANT ELEVATION
9. SUMMARY AND CONCLUSION

This analysis of track twist is a quasi-static analysis. The vertical forces were determined by a static analysis of the suspension. The lateral wheel rail forces as a function of curvature were derived from the results of a previous study. By using Nadal's Limit to determine the L/V ratio at which wheel climb will occur, a minimum vertical force necessary to support the lateral force in a curve can be determined. Once the vertical force is known, the difference in crosslevel between truck centers can be determined from the static analysis of the suspension. This difference in crosslevel between truck centers can be normalized to a difference in crosslevel in 31 feet, which is the usual measure of track twist, by assuming that the track twist is uniform.

The analysis shows that light cars with torsionally stiff bodies, such as tank cars, are more susceptible to wheel unloading due to track twist, and are consequently more susceptible to wheel climb derailments in curves due to twist, than cars such as flat cars without bulkheads, which are heavier and have more flexible car bodies. Since higher lateral forces are required in tighter curves, derailments due to track twist are more likely to occur in a tight curve.

The analysis also shows that the snubbers can have a great impact on the car's reaction to track twist. High snubber friction, or locked snubbers, can effectively lock the suspension, and so only a small amount of track twist may cause a significant wheel unloading.

Due to the geometry of the bolster, where the carbody and truck meet, a wheel cannot completely unload due to track twist. It is possible for the centerplate to separate and consequently for the truck to separate from the carbody, if the track twist is great enough.

In the analysis of the vertical loads carried by the wheels, the lateral forces acting on the truck were neglected. This does not affect the results significantly, even for several inches above or below balance speed. What will have a significant impact on the car's reaction to the track is if the curve is not smooth, but has irregularities. The response of the freight car to such irregularities can be highly dynamic in nature and cause both the lateral and vertical loads to vary over a wide range. The rock and roll phenomenon is an example of this.
APPENDIX: EFFECT OF SNUBBER FRICTION ON CAR'S REACTION TO TRACK TWIST

Snubber Force

The snubber forces can vary over a range and for almost any situation that the car could be in each snubber force can be anything from +f, helping to support the car with the maximum available friction force, to -f, trying to keep the springs compressed with the maximum available friction force, the exact value being dependent upon the recent history of the car. Several different scenarios, involving extremes of the snubber forces and of the recent history of the car can be envisioned. The six different scenarios, which bound the possible extremes of the snubber friction, have been considered. For each of these scenarios the center of gravity is assumed to coincide with the geometric center of the car floor.

Scenario 1.

In the first scenario, the car starts on level track and the snubbbers are neutral, exerting no force. The car then moves along track that evenly twists at a gradually increasing rate. As the car begins to move along the track, the snubbbers keep the suspension locked until the reaction at the spring/snubber group has changed enough to break the snubbbers out. Any compliance of the car to the track twist is due solely to the torsional flexibility of the carbody. If the car is light enough, the snubbbers may not break out until after the carbody has 'free twisted' and has rotated onto the sidebearings. Once the snubbbers have broken out, the suspension can react to the track twist. Eventually, the track twist will become great enough so that the carbody will separate from the truck centerplate.

The change in load carried by the spring snubber groups is

\[
\Delta R = \frac{KZ - (f_1 - f_2 + f_3 - f'_4)}{4(1 + \varepsilon)}
\]

As the car starts to move down the twisted track, the snubber force will be equal to the change in force, up until the change in load carried by the spring/snubber groups is greater than the maximum snubber friction force, f.

\[
f_1 = -\Delta R \quad (A-2a)
\]
\[
f_2 = \Delta R \quad (A-2b)
\]
\[
f_3 = -\Delta R \quad (A-2c)
\]
\[ f_4 = \Delta R \]  

(A-2d)

Substituting the relations for the snubber forces, equations (A-2a - d), into equation (A-1) leads to

\[ \Delta R = \frac{K_c Z}{8AB^2} \]  

(A-3)

The reaction at corner three is then

\[ R_3 = \frac{W}{4} - \frac{K_c Z}{8AB^2} \]  

(A-4)

This equation is valid until \( \Delta R \) exceeds the maximum snubber friction force. The equation indicates that only the carbody deflects in response to track twist and that the spring/snubber groups are lacked. Similar equations can be derived for the rest of the corner reactions.

When \( \Delta R \) has exceeded the maximum snubber friction, \( f \), the reaction at corner 3 changes to

\[ R_3 = \frac{W}{4} - \frac{KZ}{4} \frac{+ f}{[1 + \zeta]} \]  

(A-5)

The transition from equation (A-4) to equation (A-5) may occur before the car begins to rock on the edge of the centerplate, or after the carbody comes into contact with the side bearings.

**Scenario 2.**

In the second scenario, the car again starts on level track, but in this scenario all the snubbers are exerting the maximum available friction force to help support the carbody, +\( f \). Again the car moves along track that evenly twists at an increasing rate. Only two snubbers remain locked when the car begins to move along the track. This is because two diagonally opposite snubbers are exerting forces in the same direction as the change in the spring/snubber forces, while the other two snubbers are exerting forces in the opposite direction as the change in the spring/snubber group forces. The snubber force must change from one extreme, +\( f \), to the other extreme, -\( f \). This will not occur until the spring/snubber group reaction has changed by twice the maximum snubber force, 2\( f \). Once the snubber have broken out, the suspension can react to the track twist, and eventually the track twist will become great enough so that the carbody will separate from the truck centerplate.
The change in the spring/snubber group reactions is the same as it is for the previous case (equation A-3) but the snubber forces are different. Two of the snubbers are exerting their forces in the same direction as the change in force, \( f_1 \) and \( f_3 \), but the friction forces exerted by the other two snubbers must reverse direction. The forces exerted by the snubbers are then

\[
\begin{align*}
  f_1 &= -f \\ 
  f_2 &= -f + \Delta R \\ 
  f_3 &= -f \\ 
  f_4 &= -f + \Delta R
\end{align*}
\]

Substituting these equations for the snubber force into the equation for \( R \), equation (A-3), leads to

\[
\Delta R = \frac{KZ}{2 + 4\epsilon}
\]

The reaction at the spring/snubber group at corner three is then

\[
R_3 = \frac{W}{4} - \frac{KZ}{2 + 4\epsilon}
\]

When \( f_2 \) and \( f_4 \) become equal to \( +f \), equation (A-3) governs, the same as in the case where the friction starts at zero. \( f_2 \) and \( f_4 \) will become equal to \( +f \) when \( R \) has changed by twice the breakout friction of the snubbers.

**Scenario 3.**

The third scenario is similar to the second scenario, except that the snubbers are exerting the maximum available friction force to keep the springs compressed, \(-f\), rather than a force helping to support the carbody. The reaction of the car to track twist is similar to its reaction in the second scenario. Two spring snubber groups remain locked until the reaction at these spring/snubber groups changes by \( 2f \), but these are the opposite spring/snubber groups from the ones that remained locked in the second scenario.

The equations for the reactions as a function of twist are the same as for the previous case. If the snubber forces are assumed to start in the opposite direction, helping to support the carbody, \( +f \), the same results for the reactions are obtained. What happens is that \( \Delta R \) now affects \( f_1 \) and \( f_3 \). Substituting this into the friction terms of the equation for \( \Delta R \), equation (A-1), produces equation (A-7).
Scenario 4.

In the fourth scenario, the car again starts on level track and travels on track that increasingly twists. The snubbers at corners 2 and 4 are exerting the maximum friction force to help support the car, while the snubbers at corners 1 and 3 are exerting the maximum friction force in the opposite direction, trying to keep the springs compressed. The two snubbers for each bolster are exerting forces in the opposite directions. For light cars, this tends to cock the two bolsters in opposite direction. Because of this, an amount of 'free twist' may already be taken up. Once the 'free twist' has been taken up, the track twist is reacted to by all the suspension elements. Since the snubber friction forces do not change direction, there is no point where the suspension is locked.

If the snubbers are able to exert a force that is greater than 18% of the nominal load carried by the spring/snubber group, then the bolsters will be cocked with respect to each other when the car is on level track. If the car is sitting on level track, the change in the loads carried by the spring/snubber groups form W/4 cannot be greater than 18%. If the car is sitting on level track and this is true, then what has happened is that the 'free twist' has cancelled the twist seen by the suspension. It is likely that the carbody has rotated about the edge of one centerplate and onto one side bearing and is resting on the edge of the centerplate of the other truck. The minimum weight for this not to happen is given by

\[ \frac{W}{4} \left( \frac{W}{4} + 4 \right) < 1.18 \]

q.e.d. \( w > 89 \) kips

If the weight of the carbody is less than 89 kips, then the amount of 'free twist' must be equal and opposite to the twist that is absorbed by the suspension. This can be determined from the equation

\[ \frac{0.82W}{4} = \frac{W}{4} + f \left( \frac{KZ}{4} \right) \]

\[ Z_1 = \frac{0.18(1+\xi)}{K} - 4f \]

(A-9) (A-10) (A-11)
The amount of 'free twist' that is still left is

\[
T = \frac{4H\Delta_{SB}}{r_{SB} - r_{cp}} - \frac{H}{B}Z_i
\]  

(A-12)

After the remaining 'free twist' has been taken up, the carbody will come into contact with the second side bearing. Since the snubber forces were initially oriented in the proper direction, the load carried by spring/snubber group 3 is given by

\[
R_3 = \frac{KZ}{4} + f
\]

\[
R_3 = \frac{W}{4} - \frac{4}{[1 + \xi]}
\]

This equation applies after the free twist has been taken up.

**Scenario 5.**

The fifth scenario is similar to the fourth scenario, except initially, when the car is on level track, the snubber forces are oriented in the opposite direction. The snubbers at corners 2 and 4 are exerting the maximum friction force to keep the springs compressed, while the snubbers at corners 2 and 3 are exerting the maximum snubber force to support the car. The bolsters are again cocked, but in the opposite direction as the previous case. The carbody may have started to rotate about the opposite edge of the centerplate. The snubber forces must reverse direction, and so the suspension remains locked until the reactions at the spring/snubber groups has changed by \(2f\).

The carbody will have started to rotate about the edge of a centerplate for the same criteria as in the previous case, if the carbody weighs less than 89 kips.

Since the snubber forces are oriented in the opposite direction as the previous case, the bolsters are cocked in the opposite direction. If the carbody weight is less than 89 kips, the car has started to rotate about the edges of the centerplates, but in this case, as the car travels over the twisted track, the carbody will rotate back onto the centerplates. There is no change in the loads carried by the spring/snubber groups until the carbody has come into full contact with the centerplates. The amount of available 'free twist' is given by

\[
1.18W = \frac{W}{4} - \frac{4}{[1 + \xi]}
\]

\[
Z = \frac{KZ}{K} - 1.18[1 + \xi]W - 4f
\]  

(A-14)

(A-15)
Once the car has rotated back onto the centerplates, the loads carried by the spring/snubber groups will begin to change. Since the snubber forces must reverse direction, the suspension remains locked until the loads change by $2\ell$. The load carried by the spring/snubber group at corner is given by

$$R_3 = \frac{W}{4} - \frac{KZ}{8AB^2} - \frac{\ell}{1+\xi}$$  \hspace{1cm} (A-16)

until $R_3$ has changed by $2\ell$. The carbody will start rotating about the opposite edges of the centerplates, when $R_3$ has decreased from $W/4$ by 18%. This may occur before or after the snubbers have broken out. The carbody will rotate through the full amount of free twist given by equation (2-22). Once the snubbers have broken out, the load carried by the spring/snubber group at corner 3 is given by

$$R_3 = \frac{KZ}{4} + \frac{\ell}{1+\xi}$$  \hspace{1cm} (A-17)

The carbody will separate from the centerplate at 65% unloading.

**Scenario 6.**

In this scenario, the car starts on track that is twisted by the maximum amount, the track then evenly twists in the opposite direction. In this way, the car goes from maximum twist in one direction through to the maximum twist in the opposite direction.

Initially, when the car is on the track that is twisted to one extreme, the spring/snubber group at corner 3 is overloaded by 65%. The weight of the carbody is carried by two sidebearings. The amount of initial twist, $Z^*$, necessary for this to occur can be determined from

$$1.65W = \frac{W}{4} - \frac{KZ^*}{4} - f - \frac{f}{1+\xi}$$  \hspace{1cm} (A-18)

$$Z^* = -\frac{.65(1+\xi)W-4f}{K}$$  \hspace{1cm} (A-19)

Since the snubber forces must reverse direction, the track twist is reacted only by the carbody. Until the load carried by the spring/snubber groups have changed $2\ell$, the load carried at corner 3 is given by

$$R_3 = \frac{W}{4} - \frac{f}{1+\xi} + \frac{K\ell Z}{8AB^2}$$  \hspace{1cm} (A-20)
At 18% overload of $R_3$, the carbody begins to rotate back onto the centerplates. This may or may not occur before the snubbers break out, depending upon the weight of the carbody. The carbody will begin to rotate towards the opposite sidebearings when $R_3$ has decreased from $W/4$ by 18%. After the snubbers have broken out, the load carried by the spring/snubber group at corner 3 is given by

$$R_3 = \frac{W}{4} - \frac{K(Z+Z^*) + f}{4[1+\xi]}$$

(A-21)

Again, the carbody will separate from the centerplates when $R_3$ has decreased by 65% from $W/4$.

Using this notation, of $Z^*$, the initial track twist appears to be zero. To properly compare the results for this scenario with the results of the other scenarios, the term $(Z+Z^*)$ must be compared with what is $Z$ in other scenarios.

Figure A-1 shows the spring/snubber group vs. the difference in croslevel between truck centers for a 100 to unloaded covered hopper car in each of the six different scenarios. The final scenario (scenario 6), where the car goes from track that is twisted to one extreme, is the worst case.

**FIGURE A-1. SPRING/SNUBBER GROUP LOAD VS. SPRING/SNUBBER GROUP DISPLACEMENT AT CORNER 3**
REFERENCES


