Freight Car Dynamics
Final Report

E. H. Law
N. K. Cooperrider

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*T 1 in. = 2.54 cm (exactly). For other exact conversions and more detail tables see NBS Misc. Publ. 286. Units of Weight and Measures. Price $2.25 SD Catalog No. C13 10 286.*
EXECUTIVE SUMMARY

The objective of this research project was to develop techniques to analyze the lateral dynamic behavior of railroad freight cars. The effort included development and correlation of theoretical techniques for predicting freight car dynamic behavior, and use of the techniques to investigate the behavior of present and proposed designs. The project was sponsored by FRA with support and cooperation from the Association of American Railroads and the Union Pacific Railroad.

The models and analysis techniques developed can be used to examine causes of present dynamic problems, and to design components and vehicles that alleviate such problems. Models and analyses have been developed for representing the wheel-rail interaction mechanics and for determining freight car stability and forced response.

In the course of the work several innovations were introduced into rail vehicle analysis and testing including development of numerical techniques to deal with arbitrary wheel and rail head profiles, the use of quasi-linearization techniques to handle nonlinear characteristics, the use of an hydraulic excitation system during the vehicle tests, and the application of the random decrement technique in analyzing test data.

A number of models and analysis approaches were developed for freight car behavior on tangent track. These models differ widely in complexity. An output of the project is the understanding of the appropriate use of each model and analysis technique.

Extensive field tests were planned and carried out with the cooperation and support of the Association of American Railroads (AAR), the Union Pacific Railroad (UP), and Martin Denver Division. These tests were conducted by the AAR and UP. Eight different vehicle configurations were tested, each at several speeds on both tangent and curved track. Data obtained from these tests were used for comparisons with theoretical predictions of vehicle response.

On the basis of the limited comparisons made between test and theory, it seems evident that purely linear models, analysis, and dynamic response criteria are unsuitable for representing accurately actual freight car lateral response due to the dominance of the nonlinear vehicle characteristics. Nevertheless, these linear models and analyses are quite useful in providing an understanding of the trends in response produced by vehicle parameters, operating conditions, and environment. Both the statistical linearization approach and the hybrid simulation of the nonlinear vehicle show considerable promise in more accurately representing the nonlinear behavior of the actual vehicle. However, due to unknown parameters such as creep force characteristics and the unknown forcing due to track crosslevel, alignment, and wheel/rail offset terms, it cannot be said that any of the models were strictly validated.
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Chapter 1

INTRODUCTION

This final report summarizes the "Freight Car Dynamics" project conducted by Clemson and Arizona State Universities in cooperation with the Association of American Railroads (AAR) over the period December 1975 to April 1981. The overall objective of this project was to develop tools and techniques for the analysis of railroad freight car dynamics.

The primary motivation behind this project was the poor dynamic performance of conventional North American freight cars with 3-piece trucks. It is widely recognized that vehicles of this type frequently experience self-excited, "hunting" oscillations, encounter severe wheel wear, and are often involved in derailments. A theory for the dynamics of these vehicles was needed to understand the causes and cures for these problems.

At the time the project began, a linear theory for the lateral dynamics of rail vehicles had been developed and applied with some success to the analysis of rail passenger cars [1]. However, only one effort had been made to apply this theory to the North American freight car [2]. Thus one objective of the project was to develop theoretical tools for analyzing freight car dynamics. The use of these analytical tools to investigate and evaluate design modifications or new operating practices was the ultimate goal of this effort.

The existing theory for rail vehicle lateral dynamics also had not been quantitatively validated by comparison with experimental results, although qualitative comparisons with experimental results for rail passenger cars indicated agreement. The most complete effort, prior to this project, to make a quantitative comparison had limited success [3], primarily due to the complexities and uncertainties of the wheel/rail contact mechanics. A second objective of this project was to undertake another comparison of theoretical and experimental results, in this case using a rail freight car.

At the outset, we believed that the project involved a straightforward application of the available linear theory to the unique geometry of the 3-piece freight truck and a limited test program to obtain data for comparison with theory. We learned that the problem was considerably more complex.

One important result of this effort has been the recognition of the strongly nonlinear nature of the 3-piece freight truck, and the high level of variability and uncertainty in the dominant parameters of the vehicle/roadbed system. Consequently, the simplified plan that we began with in this project was modified several times to accommodate our increasing knowledge about the nature of the problem.

PROJECT SUMMARY

We quickly learned that additional effort in modeling the wheel/rail contact mechanics was needed. Thus, early in the project, a technique for modeling arbitrary wheel and rail cross-sectional geometries was developed, and computer implementations of existing theories for wheel/rail contact forces were completed.

We also found that the suspension connections between components of the 3-piece truck were strongly nonlinear, and in some cases were not intended by design to affect the vehicle dynamics. As a result, a single model could not be developed, but rather models of varying complexity were needed to investigate the influence of these interconnections and to determine the most suitable models for design and evaluation purposes. The nonlinear nature of the vehicle system also necessitated use of several solution techniques to find methods that offered acceptable accuracy with reasonable cost in time and money.

We quickly learned that we faced additional complexities on the experimental side as well. The vehicle suspension connections could not be determined by simple analysis or experiment, and extensive tests were required to establish these parameters. The wheel/rail geometry and wheel/rail contact force parameters also required careful measurement.

The field tests to provide experimental data were conducted by the AAR and the Union Pacific (UP) Railroad during late fall and early winter of 1976-1977.

The bulk of the data analysis and comparison with theory was performed in the fall of 1977 and spring of 1978. Unfortunately, the project funds were expended before we could complete a thorough comparison of theory and experiment.

Nevertheless, this project resulted in significant accomplishments. A series of theoretical models, associated solution techniques, and computer programs for predicting freight car lateral dynamic response were developed. Linear, quasilinear and nonlinear analyses of these models for freight car behavior on tangent track were implemented. A comprehensive field test program was planned and carried out including vehicle and roadbed characterization, extensive vehicle instrumentation, and operation of the test vehicle in several configurations and test modes. The data from these tests were reduced and analyzed by a variety of means, some of which had not been used in rail vehicle testing prior to our application. Limited comparisons of theory with experimental results were carried out, and numerous areas for future investigation and development identified.

In the course of this project, several innovations have been introduced into rail vehicle dynamics analysis and testing. These include development of numerical techniques to deal with arbitrary wheel and rail head profiles, the use of quasi-linearization techniques to handle nonlinear vehicle characteristics, the use of a hydraulic excitation system during the vehicle tests, and the application of the random decrement technique to analyze the test data.
Unfortunately a number of efforts in addition to the comparison of theory and experiment remain uneompleted at the termination of the project. The most important of these are a thorough analytical study of the nonlinear behavior of the freight vehicle; completion of a nonlinear, steady state curving theory for the freight car; and the development of methods for identifying the wheel/rail force conditions that prevail during vehicle operation.

The remainder of this report provides an overview of the results from this effort in the areas of vehicle modeling, vehicle analysis, and field testing and comparison with theory. The formal and informal project reports and the computer program users' manuals produced during this project are listed and described in the appendices.
Chapter 2

RAIL VEHICLE MODELING

INTRODUCTION

The mathematical models developed in this project for rail freight car dynamics were intended for use in addressing a variety of concerns including stability, riding quality, component wear, and the magnitude of the wheel/rail contact forces. These models were also intended for use in different applications ranging from questions of vehicle design to matters of safety. Consequently, a hierarchy of models has been developed for use in different situations and to answer different questions.

This hierarchical approach was particularly important for a system as nonlinear as the North American freight car with its three-piece trucks. Conventional analysis methods for complex nonlinear systems (i.e., direct integration of the equations of motion using digital or analog computers) are quite expensive, time consuming, and yield diffuse results in the form of time histories of the system variables. A hierarchy of models with varying degrees of detail and solution techniques that include linear and quasi-linear methods permits cheaper computation and more condensed results. The performance predicted by these different models and solution techniques is being compared with field test results to establish the conditions and range of applicability of each model and analysis approach. Although this evaluation has not been completed in this project, it eventually will allow the vehicle designer or engineer to select the model and computer program that is most appropriate for his application.

The unique dynamic behavior of rail vehicles arises from the interaction of the wheel and rail. Adequate representation of freight car dynamics for arbitrary conditions of wheel and rail wear, vehicle speed, and roadbed geometry required development of tools for analyzing the wheel/rail forces. This, in turn, necessitated analysis of the wheel/rail geometric interaction as well as application of available theories for the relationship between tangential wheel/rail creep forces and the corresponding relative velocities or creepages.

The hierarchical approach was then followed by putting together equations of motion of varying complexity to describe the freight vehicle dynamics. Our accomplishments in each area of vehicle modeling are summarized below.

WHEEL/RAIL GEOMETRY

The parameters that characterize the geometry or kinematics of wheel and rail contact have a dominant influence on rail vehicle dynamic behavior. The most important of these parameters, or wheel/rail geometric constraint relationships, are those describing the wheel rolling radii and the wheel/rail contact angles as functions of the wheelset lateral position. Combinations of these functions enter the rail vehicle equations of motion in the terms that are referred to as effective conicity and gravitational stiffness.

The wheel/rail constraint functions can be estimated or computed for idealized wheel and rail profiles such as simple conical sections, cylinders, or profiles with constant transverse radii of curvature. These estimates have been used exclusively to study the small motions of vehicles with new or slightly worn wheels.

In developing mathematical models for the prediction and evaluation of freight car dynamic behavior, it became apparent that idealized wheel and rail profiles and associated wheel/rail constraint relationships cannot be used in most cases. The worn wheel profiles observed on many of the freight cars in service appear to exhibit severely nonlinear characteristics that we expect to have a dominant influence on the vehicle stability and motion. Consequently, we undertook development of a method for obtaining accurately and rapidly the desired wheel/rail constraint relationships for arbitrary wheel and rail head profiles.

PROBLEM DESCRIPTION

Linear estimates of the wheel/rail geometric constraint functions can be easily obtained when the wheels are modeled with conical tread. However, the dominant influence of worn or profiled wheels is not represented by such a model. A better representation models the wheel and rail by circular arcs [4], but this approach is only valid for small ranges of wheel/rail motion and is not easily linearized. Numerical techniques that treat arbitrary wheel and rail geometry have been developed by European railroad administrations [5], but these techniques are not available for general use.

Consequently, we decided to develop a general procedure for calculating the required wheel/rail geometric constraint functions.

Our specific objective was to develop the capability of determining the wheel/rail geometric constraints for arbitrary wheel, rail, and track structure conditions, and to put these relationships in a form that could be easily incorporated into rail vehicle dynamic analyses. We felt that achievement of this objective required development of analytical techniques to determine numerically the desired relationships, conduct of experiments to determine the relationships for several representative wheel and rail conditions, validation of the analytical technique with the experimental results, and computation of the constraint relationships for a limited sample of cases.

APPROACH

Railway wheelsets, as they roll along the track, are constrained to move laterally and vertically in a prescribed space determined by the geometry of the wheels, rails and track structure. The characteristics of these geometric constraints determine, to a large extent, the nature of the lateral motions of the wheelsets. The wheelset position may be described by two independent variables, the lateral position of its geometric center relative to the track centerline, \( x_w \), and the angular rotation of the wheelset about a vertical axis, \( \theta_w \). The remaining motions of the wheelset such as roll or vertical movement, are determined by the geometric constraints.

For our purposes in studying the lateral dynamics of rail vehicles, we must know the following
information as a function of the independent variables, $x_w$ and $y_w$:

$$r_L = \text{instantaneous rolling radius of the left wheel}$$

$$r_R = \text{instantaneous rolling radius of the right wheel}$$

$$y_L = \text{instantaneous height of contact point on the left rail}$$

$$y_R = \text{instantaneous height of contact point on the right rail}$$

$$\delta_L = \text{angle between the contact plane on the left wheel and the axle centerline}$$

$$\delta_R = \text{angle between the contact plane on the right wheel and the axle centerline}$$

$$\phi_w = \text{roll angle of the wheelset with respect to the plane of the rails.}$$

These constrained variables, corresponding coordinate systems, and contact point definitions, are illustrated in Figure 2-1.

![Fig. 2-1 Wheel/Rail Parameters, Rear View](image)

The dependence of these constrained variables on the yaw angle of the wheelset is a second order effect. Consequently, for our purposes it is sufficient to determine the functional relationships between the wheelset lateral displacement and each of the constrained variables.

The approach we took to modeling the rail-wheel geometry can be broken down into the following steps:

1. Formulation of mathematical descriptions of the rail and wheel profiles using a series of fourth-order polynomials over sub-intervals of the profile.

2. Calculation of the locations of the contact points.

3. Calculation of the desired parameters using the contact point locations and the polynomial descriptions of the wheel and rail profiles.

4. Computation of describing functions, or quasi-linear functions to represent the resulting relationships.

Our approach to describing mathematically the wheel and rail profiles involved fitting a series of fourth-order polynomials to the tabular wheel and rail data. Curve-fitting was used rather than interpolation between the data points because it provided some numerical smoothing of irregularities in the input data. The polynomials also allowed easy calculation of the slopes of the profiles. A series of fourth-order curves was used rather than fewer higher-order curves or a different function because of the reduced complexity of calculations necessary to manipulate polynomials of only fourth order.

We calculated the contact point locations by applying a numerical search procedure to find the locations where the difference in height between the wheel profile and the rail profile is a minimum.

The geometric constraint relationships were computed by substituting the profile equations and the computed contact point locations into the defining equations for the constraint relationships on a point-by-point basis. For example, the rolling radius of a wheel at a specified wheelset lateral displacement was found by substituting the contact point location into the corresponding wheel profile equation, a fourth-order polynomial in the appropriate interval, to obtain the rolling radius at the given contact location.

Quasi-linear descriptions were needed for some of the wheel/rail geometric constraint relationships. The describing function technique with sinusoidal and random inputs was used to compute quasi-linear functions for the difference in rolling radii, the difference in contact angles, and the wheelset roll constraints.

The computational procedures for this wheel/rail contact analysis were programmed in FORTRAN. The resulting program, titled WHIRAIL, is described in [6]. This report also discusses the successful comparison of the analysis with experimental data, and contains a limited exploration of the effects of wheel and rail wear, wheel profile contour, and rail gauge on the wheel/rail geometric constraint functions.

In order to better understand the information provided by this analysis, it is useful to study a typical result generated by the WHIRAIL program. Figure 2-2 illustrates the contact positions and wheel/rail constraint relationships for 1/20 taper conical wheels on a worn rail head at nominal (56.5 inch) track gauge. The contact positions on the wheel and rail shown in this figure illustrate that the contact position on the rail remains nearly stationary at a position just to the inside of the rail center until the flange contacts. Consequently, over this range the wheel contact point moves in a direction opposite that of the wheelset lateral displacement and nearly equal in magnitude. The one slight jump in contact point at a lateral displacement of about 0.27 inches is probably due to a spurious dip in one of the profiles introduced either in the data generation process or during the curve fit procedure. This slight dip has very little effect on the resulting constraint relationships, as the curves in Figures 2-2c, d, e, f, and g illustrate.

As expected, all the constraint relationships are nearly linear over the range before the flange contacts the rails. When the flange contacts the rail, the contact point moves to the inside edge of
the rail. The contact point on the wheel also jumps from the tread to the flange. This contact jump is reflected in jump discontinuities in the rolling radii difference and contact angle difference. Further lateral displacement of the wheelset moves the contact point on up the flange and eventually down the other side when the displacement exceeds 0.70 inches.

The WHRAIL computer program deals with the case where the left and right wheels and rails are mirror images of each other. A second program, WHRAILA, was developed that relaxes this condition and allows asymmetric wheels and rails. Additionally, a technique utilizing cubic splines for calculating the curvature of the wheel and rail is incorporated in WHRAILA. This information is needed for calculating the creep coefficients and nonlinear creep force-creepage relationships. The WHRAILA program and its usage are described in [7].

A technique was also developed in this project for fast and efficient digitization of graphical wheel and rail profile data [8]. These digitized data are needed as input to the wheel/rail constraint programs.

**WHEEL/RAIL CONTACT FORCES**

The shear stresses acting between wheel and rail in the contact region give rise to creep forces and moments. These stresses and corresponding forces and moments are functions of the relative wheel/rail velocity. When normalized by the forward speed, the relative velocity is termed the creepage, and its components in the rolling and transverse directions the longitudinal and lateral creepages.

The creep force-creepage functional relationship is an essential element of a rail vehicle dynamic model. Unfortunately, the nature of this function is not well established. Estimating the parameters associated with the creep force laws is one of the most difficult and uncertain steps in assembling the system parameters needed to analyze rail vehicle dynamics.

**Problem Description**

Kalker's nonlinear theory of creep [9,10] is regarded as the most complete and accurate theory available. The nonlinear creep force characteristics given by this theory compare well with laboratory experiments [11]. In addition to predicting the longitudinal and lateral forces due to relative longitudinal and lateral velocities between wheel and rail, Kalker's linear and nonlinear theories are the only ones among all the available treatments that account for the dependence of the lateral creep force on the relative angular velocity, or spin, between wheel and rail. This force compon-
ent, termed the lateral-spin creep force, is usually less than the lateral creep force component due to lateral creepage when the wheel/rail contact remains on the wheel tread. However, as the contact point moves towards the flange and the wheel/rail contact angle increases, the lateral-spin component becomes dominant.

It should be noted that despite the general acceptance of Kalker's theories, they have some shortcomings when applied to rail vehicle dynamics. It has been observed that in the field the creepages corresponding to a given creep force may be considerably larger than those given by the theory, and that these relationships vary with wheel and rail surface conditions and with the environmental state. Caution should be exercised in using these theories in dynamic analyses. It is best to carry out analyses for a range of the uncertain parameters of the theory.

Our objective in this project was to put the theories of Kalker in a form that could be easily used by vehicle dynamicists.

Approach

A computer program for calculating the linear creep forces and moments of Kalker's linear theory was prepared and made available to those investigators requesting it [12]. This program provides the linear coefficients, or creep coefficients, that relate the creepages and creep forces. These coefficients are functions of the wheel and rail geometry, the wheel/rail normal force, and the material properties. In most cases these creep coefficients are used as input parameters for linear vehicle dynamic analyses.

In addition, the program implementing the "Simplified Theory of Rolling Contact" was translated from the original ALGOL to FORTRAN [13]. A significant number of changes were made in the program for more convenient use, but the fundamental equations remain unchanged.

The "Simplified Theory" program gives an appropriate solution for the resultant tangential creep forces and spin moment acting between two bodies of uniformly elastic material properties. Assumptions corresponding to the Hertz contact theory are implied, and two additional simplifying assumptions are made resulting in a significant reduction in computation as contrasted with the "Exact Theory."

Two separate computer codes were developed, the first a general solution with extended input and output, and the second a shortened version primarily intended for use as a subroutine.

In order to make a complete set of Kalker’s theories available, the computer program titled, "A Programme for Three-Dimensional Steady State Rolling" was also converted from the original ALGOL to FORTRAN [14]. This program concerns the same problem as the simplified theory except for the extension to unequal materials in the two bodies. However, the solution does not employ the simplifying assumptions of [13]. It should be noted that for equal materials both programs give nearly the same results, and in addition the simplified theory reduces the computation time by a factor of 50 to 100.

**Coulomb Friction**

The damping in suspensions of North American freight cars is provided entirely by Coulomb friction. Coulomb friction is also present at other interfaces between vehicle components such as the car body-bolster centerplate surface and the side-frame-bearing adapter-bearing surfaces. This friction can strongly influence dynamic behavior, and consequently must be accurately represented in computer analyses of rail vehicle dynamics.

In linear vehicle dynamic models, Coulomb friction is represented by equivalent viscous damping computed using quasi-linearization methods. Quasi-linear dynamic analyses obviously employ the same equivalent linearization techniques. Direct integration of the equations of motion by digital or numerical means, however, requires an approximation to the discontinuous Coulomb friction law to resolve the problem of causality when the relative velocity at the friction surface is zero.

We undertook a small study to develop an accurate simulation model of the Coulomb friction non-linearity for use in digital and analog/hybrid simulations of multiple degree-of-freedom systems [15]. Three different friction simulation models were analyzed:

1. A comparator and electronic switch model implemented on the analog computer.
2. A linear viscous band model implemented on both the analog and digital computers.
3. A slider model implemented on the digital computer.

The performance of these models in a simulation of a one-degree-of-freedom, harmonically forced spring-mass-damper system was studied.

The results of this study indicated that for an analog simulation, the comparator model is most accurate. The slider model proved to be most accurate for digital simulation. These results were utilized in our later vehicle simulation efforts.

**Vehicle Models**

The wheel/rail geometric constraints and the creep force laws described above were incorporated into several different mathematical models for the lateral dynamic behavior of a single railway freight vehicle. Our attention in this project was focused on the lateral behavior because it gives rise to most of the serious problems experienced with railway freight cars. Hunting, rock and roll, wheel and rail wear, gauge spreading and wheel climb, for example, can be attributed to poor lateral vehicle dynamics. Vertical motions are only weakly coupled through nonlinearities or asymmetric construction to the lateral motions and have been neglected to reduce model complexity. It is quite easy to study vertical dynamics independently with models that are relatively straightforward to develop. Lateral dynamics, in this context, account for the lateral, yaw and roll motions of the various vehicle components.

A series of models for the lateral dynamics of a single railway vehicle was developed. These are described in Table 2-1, classified by the number of
degrees of freedom in each. The equations for wheelset motions are identical in these models. The differences arise in the assumptions concerning the number of additional vehicle components, the nature of their interconnections, and the possible component motions.

Nonlinear equations of motion can be written for each model in Table 2-1. A number of analyses, or solution methods, are available to solve these equations. In this project, solutions were found by linearization and subsequent analysis, by quasi-linearization and subsequent analysis, and by direct integration on a hybrid computer. The development of the equations of motion for these models is documented in [16,17]. These documents also address the analysis of the equations and illustrate the application of the models and analysis to various rail vehicle problems.

As noted earlier, one of our objectives in this project has been to develop and identify the simplest credible model and analysis technique consistent with its ultimate use. The available combinations of models and analysis techniques and their role in addressing important rail vehicle dynamic problems is discussed further below.

Hunting Stability

One of the most severe problems facing the railroad industry today is that of ensuring that the various rail vehicles in service have an adequate margin of safety with regard to hunting stability. Unfortunately, the practical solution of the hunting problem for all railway vehicles is not yet here. However, the theoretical models and analysis techniques developed in this project should enable rail vehicle designers to develop new vehicles that offer sufficient safety against hunting, and find corrective measures for many existing vehicles.

Linear Analyses

Eigenvalue-eigenvector stability analyses have been developed for linearized versions of each of the models of Table 2-1. As discussed previously, the presence of Coulomb friction in the vehicle suspension requires quasi-linearization of this nonlinearity on a component basis in order to linearize the equations of motion. These eigenvalue analyses predict the frequency and damping of oscillatory modes and the time constants for overdamped modes as well as the shape of each mode of vehicle motion. This information permits the estimation of stability margins from the variation of the damping with speed. An example of these results is shown in Figure 2-3.

*This model was developed in the early stages of the research when it was thought there was a possibility of performing tests with a similar physical configuration on the Japanese National Railways (JNR) roller rig.
Computer programs implementing these eigenvalue-eigenvector analyses are documented in [18]. This user's manual also addresses the use of the computer programs.

The selection of input data for the eigenvalue-eigenvector programs is not a simple matter, even when component test data are available. For vehicles such as conventional North American freight cars, the lateral suspension characteristics are dominated by dry friction and other nonlinearities such as deadband and hardening springs. The choice of an effective linear suspension representation requires prior knowledge of the vehicle dynamic environment.

The primary advantages of linear analyses are the economical computer costs and the readily understood insight into the effects of various parameters on the vehicle dynamics.

Quasi-Linear Analysis

The suspensions of typical freight cars are dominated by nonlinearities. In addition, the wheel/rail interaction is characterized by nonlinear wheel/rail geometry and nonlinear creep force-creepage relationships. These nonlinearities strongly affect the lateral dynamic response of rail vehicles.

As discussed previously, there are many uses for linearized stability analyses of rail vehicles. These should be used with considerable care and judgment when strongly nonlinear characteristics exist. For those cases where a detailed examination of nonlinear effects is desired, nonlinear analyses must be used. Because the computation costs are usually an order of magnitude greater than those of linear analyses, these nonlinear analyses should be used with discretion.

A quasi-linear analysis for hunting stability seeks to find a middle ground between linear analysis and nonlinear simulation by utilizing linear analysis techniques in a special way. Such an analysis determines the existence and stability characteristics of limit cycles. The work reported in [19,20,21] represents the first efforts to apply these techniques to rail vehicles. These results agree very well with those obtained by direct integration of the equations of motion.

Results obtained via quasi-linear analysis for the limit cycle amplitude vs. speed relationships are shown in Figure 2-4 for the 9 DOF freight car model. Unstable limit cycles may be thought of as stability boundaries, while stable limit cycles represent the hunting behavior. In this case, one sees the effect of varying wheel profiles and introducing suspension friction. We have applied the quasi-linear analysis to the 9 DOF freight car model as well as to simpler models, although it may be used for almost any nonlinear model.

The computation costs in developing curves such as those shown in Figure 2-4 via quasi-linear analysis are much less than the costs associated with numerical integration of the equations by digital computation. It should be noted, however, that quasi-linearization is difficult to apply to complicated nonlinear relationships such as Kalker's nonlinear creep theory, and that in certain situations convergence problems are encountered with the quasi-linear solution algorithms. Future research may find solutions to both these drawbacks.

Nonlinear Analysis

Solution of nonlinear equations of motion was addressed using the hybrid computer. Although fewer people have direct access to hybrid computers than to digital computers, hybrid computation can offer significant cost savings over digital integration. These questions are discussed more completely in [22].

Due to machine capacity limitations on the Clemson University Engineering Computer Laboratory hybrid computer, we have focused our hybrid computation efforts on the 5 DOF half-car model. The nonlinearities included are the wheel/rail geometric constraint functions and suspension friction. Random lateral rail alignment irregularities were also introduced to study forced response.

Results of the hybrid simulation of the 5 DOF model are shown in Figure 2-5. This plot of limit cycle amplitude vs. speed illustrates two cases...
using the same vehicle parameters. In one case equivalent viscous damping is used for Coulomb friction, while the actual Coulomb friction characteristics are used in the second case. For practical purposes, we may be most interested in the speed at point A on the curve, which corresponds to the lowest speed that can sustain hunting behavior. Point B in this figure corresponds to the highest predicted speed at which stable limit cycles, or hunting behavior, can exist. This analysis predicts derailment at higher speeds. These hybrid simulation efforts are discussed more completely in [17,22].

In summary, we have developed six different models of freight vehicles and have used three different analysis techniques (linear eigenvalue-eigenvector analysis, quasi-linear analysis, and direct integration using a hybrid computer) for evaluating the lateral stability of railway freight cars.

Forced Response

In assessing vehicle behavior, the designer and operator are also interested in acceleration, force and stress levels at various positions in the vehicle, and in displacements of suspension elements and other components. Determining such quantities requires analysis of the forced response of the rail vehicle to roadbed irregularities. Our work on forced response has concentrated on the 5 and 9 DOF models. Linear and quasi-linear frequency domain methods, and direct integration of the nonlinear equations have been employed.

Linear Frequency Domain Analysis

The simplest and least expensive analysis approach uses standard, linear system, frequency analysis techniques. These techniques yield results in either or both of two forms. In the first, the amplitudes of the vehicle response variables (displacements, accelerations, forces across suspension elements and between wheel and rail, for example) are obtained as a function of the frequency and amplitude of the roadbed alignment and/or cross-level irregularity. In the second form, the power spectral densities (PSD's) of the same vehicle response variables are found for a given roadbed irregularity PSD. Naturally, the equations of motion must be linearized in order to use these methods.

A computer program implementing this analysis for the 9 DOF model is described in [23]. Typical PSD's obtained from this program are shown in Figure 2-6. In this figure, the lead truck lateral, lead truck warp, car body lateral and car body roll PSD's in response to a roadbed alignment input are shown for vehicle parameters typical of an open hopper car. The dominant kinematic mode at about 0.5 Hz is evident in the truck motion PSD's. The car body PSD's illustrate the "dropout" phenomenon or filtering effect, due to the fact that the car body and truck sideframes are traveling chords that do not respond at their centers to certain input wavelengths.

Quasi-Linear Frequency Domain Analysis

To more accurately consider the effects of the suspension and wheel/rail nonlinearities, quasi-linear analysis was used [19]. The output characteristics of the various nonlinear suspension elements will depend on the characteristics of the input motion to the nonlinearity. The quasi-linear approach accounts for this dependency and allows the designer to calculate the influence of the disturbance magnitude and form on the vehicle response.

Both sinusoidal and random roadbed inputs were treated. In this case, the frequency response will depend nonlinearly on the amplitude of the input sinusoid, and the output PSD's will depend nonlinearly on the RMS level and shape of the input spectrum.

The application of quasi-linear techniques to the forced response of rail vehicles appears to be a very cost effective approach. Computation costs are roughly several orders of magnitude less than digital integration costs. The time and manpower required to generate and interpret results of a quasi-linear analysis also reflect similar ratios relative to digital integration methods. However, as with the quasi-linear stability analysis, there are approximations and assumptions necessary in a quasi-linear analysis that are not necessary when using a direct integration approach. Whether these pose difficulties in a given situation depends on the characteristics of the vehicle and on the information desired.

Direct Integration

We have also computed the vehicle forced response by direct integration. This work employed the same 5 DOF analysis and hybrid computer program developed for hunting stability, with the addition of the roadbed input quantities.

Results from the hybrid simulation are in the form of time histories of the vehicle response variables. These may later be processed to yield PSD's or other reduced data.

Typical results from this program are shown in Figure 2-7 for parameter values representative of an open hopper car on a randomly irregular roadbed. Note the extremely diffuse nature of the data produced by such an analysis.
Figure 2-6  Typical Freight Car Random Response PSD's

Baseline Location: On Wheel 1.2 x Marker 1.000 WMRP

Baseline Location: On Wheel 1.2 x Marker 1.000 WMRP
Figure 2-7 Hybrid Computer Simulation Results for Response Of Hopper Car at 34.3 mph to Random Alignment Irregularities
Chapter 3

FREIGHT CAR ANALYSES

INTRODUCTION

The freight car dynamic models and associated analysis techniques developed in this project have been utilized to explore questions of model sensitivity and limitations, to begin to address vehicle design and maintenance questions, and to compare theoretical vehicle results with experimental results. The latter effort, a major objective of the project, is discussed in more detail in the following chapter.

The overall objective of this project was to develop adequate models for studying vehicle dynamic behavior. As such, most of our effort in applying the models was directed toward exploring their limitations and comparing their results with experiment. Only preliminary investigation of design and maintenance questions was carried out in this project. In our opinion, this should be the focus of another project in the near future.

MODEL SENSITIVITY STUDIES

Several studies were undertaken to investigate the relative accuracy of the different models and analyses, and to study the sensitivity of the results to vehicle and roadbed parameters that we expected to be uncertain or highly variable.

Model Complexity

In order to judge the relative accuracy of the 9, 17 and 19 DOF models, a comparison of eigenvalue and eigenvector results was made [16]. This comparison indicated that the truck model used in the 9 DOF model is adequate for use in stability analyses of vehicles with roller bearing trucks. The critical speeds predicted by the 9, 17 and 19 DOF models for a vehicle with roller bearing trucks are very similar. However, the shape of the least damped mode predicted by the 19 DOF analysis differs from that predicted by the 9 or 17 DOF models in both the addition of flexible modes and in the phasing of the front and rear truck motions.

These results indicate that in the initial design stages of freight vehicles with 3-piece roller bearing trucks, the 9 DOF model may be used to investigate stability. It should be noted, however, that this study was limited to linear analyses. In situations where large motions are expected, such as during vehicle hunting, linear approximations will not be valid. We expect that more complex vehicle models will be needed for such situations.

Wheelset Flexibility

A study was completed using a linear, three degree-of-freedom, flexible wheelset model [16]. This model provided for relative angular displacements between the two wheels in addition to lateral and yaw degrees-of-freedom. The model was developed as the first "block" of a complete vehicle model. The primary objective of this study was to determine how the critical speed for the onset of hunting varies as a function of axle torsional stiffness, and thus to also determine the validity of the usual assumption that the wheelsets can be treated as rigid bodies.

Typical stability boundaries found in this study for a nominal wheelset configuration are shown in Figure 3-1. The speeds at which the real parts of the eigenvalues become positive for each of the two possible oscillatory motions of the wheelset are plotted against the nondimensional torsional stiffness, where the nominal torsional stiffness represents a 6 inch axle diameter. Note that the "rigid body" wheelset mode determines the stability boundary for torsional stiffnesses greater than 10% of the nominal, and that the critical speed of this mode approaches the critical speed for a rigid wheelset as the torsional stiffness increases. At the nominal torsional stiffness, the critical speed is about 1.7% higher than that predicted for the rigid wheelset model.

This study demonstrated that the torsional flexibility of a conventional (six inch diameter) freight car axle has a negligible effect on the critical speed.

Carbody Flexibility

An investigation employing the linear 23 degree-of-freedom freight vehicle model was conducted [16]. The effects of lateral and torsional flexibility of the car body were studied for vehicle parameters representative of an open hopper car and a flat car. The objective of this study was also to examine the effects of such flexibilities on stability, and to determine, as a result, the limits of validity of simpler models that ignore such flexibilities.

The effect of variations in car body stiffness on the stability of the hopper car configuration is shown in Figure 3-2. Over a wide range of car body stiffness the critical speed remains nearly constant at a value only a few percent less than the value predicted by the 9 degree-of-freedom model that treats the car body as rigid. However, the shapes of the least damped modes differ somewhat from those predicted by the 9 DOF model.

The effect of car body flexibility on the predicted motion and stability of the flat car config-
Critical Speed

Figure 3-2 Effect of Car Body Stiffness on Critical Speed

uration was similar to that found for the hopper car, although the loaded flat car had a somewhat higher critical speed than predicted by the 9 DOF analysis at moderate stiffness.

This study indicated that car body flexibility may be neglected for stability studies of relatively stiff vehicles.

Creep Coefficients

It is reasonably well established that the creep coefficients in the equations of Kalker's linear creep theory vary considerably with wheel and rail surface condition and with environmental factors such as humidity. It has been suggested that the creep coefficients that prevail for rail vehicle situations may be as small as 20% of those given by Kalker's theory.

Due to this uncertainty over numerical values for creep coefficients, a limited study was undertaken to determine the influence of variations in the creep coefficients on freight car stability [28]. The linear, 9 DOF freight car model was employed in this study. The parameters chosen were representative of an open hopper car, but the choice of equivalent linear characteristics required arbitrary decisions concerning amplitudes of motion across nonlinear elements.

The results of this study indicated that the creep coefficients can have a strong influence on vehicle stability and that the nature of this influence varies widely from configuration to configuration. For a freight car with conical taper wheels, we found that the critical speed of hunting was nearly proportional to the creep coefficients. Increasing all the coefficients from 50% to 100% of Kalker's theoretical values doubled the predicted critical speed. On the other hand, the critical speed for the same vehicle with hollow, Heumann profile, wheel treads decreased as the creep coefficients increased.

As a result of these somewhat surprising results, we recommended that special tests be implemented in the associated field test program in order to identify the creep relationships prevailing during the tests.

Suspension Dither

The majority of the analyses conducted in this project utilized linear models due to their ease and efficiency of use. However, one of the most significant discoveries of the project was the magnitude of the influence of Coulomb friction on the dynamic behavior of the freight car. To investigate the nature of this phenomenon we utilized the 5 DOF hybrid computer simulation [17, 30].

One of our specific interests was to determine whether higher frequency motions of the truck components influenced the friction components and, in turn, the vehicle behavior. To represent such effects, dither was introduced as a variation in the breakout force levels at the various nonlinearities. The effect of introducing dither into the simulation in each of several different vehicle configurations was to increase the lowest speed at which hunting might occur, and to increase the highest speed of stable limit cycles. In other words, the speeds corresponding to points A and B in Figure 2-4 moved to the right.

As these results produced limit cycle behavior more consistent with our field test observations, we believe that introducing dither results in a more realistic simulation.

Vehicle Design and Maintenance

The following preliminary studies of vehicle design and maintenance tradeoffs should illustrate the use and potential utility of the theory of rail vehicle dynamics for investigating such matters.

Suspension Design

To demonstrate the application of the theory in a design study, the 11 and 23 DOF models were used to examine the effects on hunting stability of various primary suspension elements [16]. In addition, a generic model of a truck with interconnected wheelsets was formulated and a range of values for the interconnecting suspension elements was examined. Typical results from this study are shown in Figure 3-3 where the critical speed for hunting instability is plotted versus the interconnection shear stiffness for a vehicle with directly interconnected wheelsets.
Asymmetric Wheelsets

These models and analyses have also been used to examine a potential maintenance problem. As a freight car accumulates service mileage, the wheels on a given truck develop different transverse profiles. A brief study was conducted to examine the effects on stability of using a different wheel profile for each of the two axles of a truck [25].

Various combinations of wheel profiles were examined. A typical result is shown in Figure 3-4 where the critical speed for hunting is shown for a nominally empty 80 ton hopper car with various wheel profile configurations. The axles labeled "N" are those with the standard AAR new profile while those labeled "P" have profiled wheels with an effective conicity of about 0.31 and a substantially increased gravitational stiffness. It can be seen that trucks with different wheel profiles on the leading and trailing axles exhibit critical speeds that depend strongly on the direction of travel.

Asymmetric Loads

The operational practice of loading freight cars asymmetrically fore and aft was also examined briefly for its effect on stability [25]. It was found that stability was increased slightly when the vehicle was loaded in the rear as opposed to the front. However, this difference usually was small relative to the difference in stability between empty and fully loaded cars.

Summary

As indicated in this brief discussion, the theory of rail vehicle dynamics can be used to answer various questions concerned with maintenance and operational practices as well as those concerned with vehicle design. The work completed in this project was intended only to provide example applications.
FIELD TESTING AND COMPARISON METHODS

INTRODUCTION

Although there has been a great deal of activity in the field of theoretical rail vehicle dynamics, there has been relatively little comparison of the theory with field test data. Few previous attempts to validate theories for rail vehicle lateral dynamics have achieved only partial success due to uncertainties in many of the system parameters such as the creep force laws, the rail head profiles, and the roadbed geometry.

In the Freight Car Dynamics project, an attempt to eliminate these uncertainties was made. Several different approaches for comparison of theoretical and experimental results were studied, associated data analysis techniques were developed, and a carefully planned set of tests was executed to provide experimental data, the data was processed and analyzed by several techniques, and results from the most promising theoretical analyses were compared with experimental results.

Although our initial objective encompassed study of freight car behavior on both tangent and curved track, deficiencies in the test conduct procedure rendered the curving data unusable for validation purposes. On curved track, validation by comparison of theoretical and experimental values of wheelset lateral displacements, wheelset yaw angles and wheel/rail contact forces in curve entry and steady curving was planned. However, the information needed to accurately locate the wheelsets relative to the rails and to obtain lateral wheel/rail forces was not obtained during the tests, and the planned comparisons could not be carried out. Consequently, our subsequent comparison effort was limited to study of behavior on tangent track.

VALIDATION TECHNIQUES

Because very little work has been done in the past to validate rail vehicle lateral dynamics theory, attention was given early in the project to alternative approaches for this task. It was evident from the beginning of the project that an adequate validation procedure entails more than comparison of single values such as the critical speed where hunting begins. Two methods for validation of tangent track theory were pursued: (1) a comparison of theoretical and experimental power spectral densities (PSD's) for several vehicle variables, and (2) a comparison of theoretical and analytical modal frequency, modal damping and mode shape vs. speed characteristics. As discussed later, both these approaches proved to have drawbacks when applied to a system as nonlinear as the North American freight car with 3-piece trucks.

In order to investigate the feasibility of using spectral analysis techniques, a comparison of theory and experiment for the vertical dynamics of a rail freight car was carried out. Experimental roadbed and vehicle dynamic response data from the TDOP tests [26,27] conducted by the Southern Pacific Railroad was used for this study. The data was processed using the spectral analysis techniques described in the next section to obtain vertical car body and truck displacement and acceleration PSD's. A linear forced response analysis of the vertical behavior of the freight car produced theoretical PSD's for corresponding values. A roadbed vertical profile PSD obtained from measurement of the test site roadbed geometry served as input to this theoretical analysis. This study, reported in [28], indicated extremely good agreement between theoretical and experimental PSD's, and demonstrated that this approach to validation should be pursued for the lateral dynamics study.

The second approach for validation of tangent track performance entails comparison of modal damping values and frequencies obtained from an eigenvalue analysis with values found experimentally. Obtaining such values experimentally for a rail vehicle on an irregular roadbed poses some problems, as discussed in the following section. However, good agreement between such values has been found for scale model rail vehicles on roller rigs [29], indicating that the approach warranted pursuit in this project.

In addition to modal damping and frequencies, mode shapes may also be compared. The theoretical mode shape information is contained in the eigenvector. Experimental mode shape information can be found by cross-spectral analysis between state variables of the vehicle system. This capability to process experimental data was developed and used.

DATA ANALYSIS TECHNIQUES

The comparison methods chosen for this project required preparation and use of computer programs for data reduction, spectral analysis, random decrement analysis and logarithmic decrement analysis. Specific accomplishments in each area are reviewed below.

Data Handling

A voluminous amount of data can easily be gathered in a series of rail vehicle field tests. The data collected in the tests conducted in connection with this project filled eleven reels of 1600 BPI magnetic tape. The task of reducing and analyzing such quantities of data can consume vast amounts of manpower and computer time, particularly if not well planned.

The first processing step entailed reading the raw data tapes; converting to engineering units; combining channels to compute the desired variables for comparison with theory; computing statistics such as mean values, standard deviations, and histograms; and plotting the time histories of selected model variables. In this effort, the raw displacement data was combined to form "model variables" for the 9 and 19 degree-of-freedom models described in Chapter 2. These experimental model variables, based on relative wheel-to-rail displacement measurements, are actually relative vehicle-to-rail lateral alignment motions.

Special purpose computer programs were written to read the mini-computer generated data tapes, convert to the "FIELDDATA" format of Arizona State's UNIVAC computer, combine channels, and convert to engineering units. The processed data was recorded on a second set of computer tapes. Programs to plot this data on a CALCOPP plotter were
was used to provide an alternative to estimation of an exponentially decaying sine wave that includes domain records. The result of this averaging process is a Random Decrement Signature that is similar to the ideal step response of the system. The Random Decrement Signature is formed by averaging a series of time-series data such as that generated in field tests.

Spectral Analysis

A spectral analysis computer program was developed to analyze digitally recorded, time series data such as that generated in field tests. This program, based on a Fast Fourier Transform algorithm, computes power spectral densities, cross spectral densities, auto correlation functions, cross correlation functions, probability densities, probability distributions, mean values, standard deviations, transfer functions and coherence functions. The processing techniques, including special considerations for spectral analysis of experimental data such as sampling rates, pre-whitening, and leakage are described in detail in [28].

The PSD's computed from experimental data were compared directly with results from theoretical random response analyses. In addition, the damping ratio and frequency of the least damped mode was estimated from the PSD's and used to compare with results of eigenvalue analyses. The damping ratio of a lightly damped mode can be estimated directly from the PSD signature by the relationship,

$$\zeta = \frac{\Delta f}{2f_0}$$

where,

- $\zeta$ — damping ratio
- $\Delta f$ — half-power bandwidth
- $f_0$ — center frequency of local peak

Three practical difficulties can occur when using this procedure. The peaks are not always well defined. Consequently, some smoothing is required. In addition, for very light damping ratios the estimates are too large because the analysis bandwidth is finite. For large damping ratios, no peak appears, and the values needed in the above estimate cannot be obtained.

Cross spectral density and transfer function analysis using this same computer program with field test data have also been used to a limited extent to estimate mode shapes, i.e., amplitude and phase angle relationships between component motions.

These methods for estimation of damping ratios and mode shapes and their application to rail vehicle dynamic test data are discussed in [30].

Random Decrement Analysis

Damping ratio and frequency estimates may also be obtained from random response data using the Random Decrement Technique [32]. This technique was used to provide an alternative to estimation of this information from PSD's. To our knowledge, this is the first application of the technique in rail vehicle dynamics. The Random Decrement Signature is obtained by averaging a series of time-domain records. The result of this averaging process is a Random Decrement Signature that is similar to the ideal step response of the system.

Once the Random Decrement Signature is formed, an exponentially decaying sine wave that includes bias and trend is fit to the signature data points to minimize the square of the error between signature and fitted function. Frequency and damping ratio are available in the appropriate parameters of the decaying sine wave.

Details of our use of the Random Decrement Technique may be found in [30].

Logarithmic Decrement Analysis

A third approach for obtaining frequency and damping ratio information was also pursued in this test program. Hydraulic actuators were mounted between the truck sideframes and the car body in order to apply a torque that would move the truck into a disturbed position relative to the rails. On release of the actuators, the transient response of the vehicle was observed and used to estimate frequency and damping of the least damped modes.

Damping ratio information is contained in the shape of the decay envelope, which is defined by the peaks in the response curve. The logarithmic decrement is defined by

$$\delta = \frac{1}{n} \ln \frac{x_n}{x_{n+1}}$$

where:

- $\delta$ — log decrement
- $x_n$ — $n^{th}$ peak value
- $x_{n+1}$ — ($n+1)^{th}$ peak value

For small damping ratios, $\delta$ is related to the damping ratio, $\zeta$, by the expression,

$$\delta = 2\pi\zeta$$

The above equations yield the following relationship between $\delta$ and $\zeta$ when $\zeta$ is small,

$$\zeta = \frac{1}{\pi} \ln \frac{x_n}{x_{n+1}}$$

TESTING PROCEDURES

The field tests to provide data for the validation effort were planned in cooperation with the Association of American Railroads (AAR) and conducted by the AAR and the Union Pacific (UP) Railroad. The tests were conducted during late fall and early winter of 1976-1977 on the UP mainline west of Las Vegas, Nevada. The test objectives, test philosophy and test requirements for these tests are discussed in detail in the program planning document [33].

The test vehicle, shown in Figure 4-1, was a Louisville and Nashville (L and N) Railroad, 80 ton, open hopper car equipped with 70 ton, Ride Control trucks. The field tests were conducted with this vehicle in eight different configurations of wheel profile, load, truck to car rotational resistance, and truck warp stiffness as shown in Table 4-1. These variations in vehicle condition were selected primarily to determine whether the theoretical analyses would correctly predict the effects of such vehicle changes on vehicle dynamic performance.
Table 4-1 Test Vehicle Configurations

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Wheels</th>
<th>Load</th>
<th>Side-Bearings</th>
<th>Truck Stiffener</th>
<th>Centerplate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CN Profile A</td>
<td>0</td>
<td>None</td>
<td>None</td>
<td>Dry</td>
</tr>
<tr>
<td>2</td>
<td>CN Profile A</td>
<td>0</td>
<td>None</td>
<td>None</td>
<td>Lubricated</td>
</tr>
<tr>
<td>3</td>
<td>CN Profile A</td>
<td>0</td>
<td>None</td>
<td>None</td>
<td>Lubricated</td>
</tr>
<tr>
<td>4</td>
<td>CN Profile A</td>
<td>0</td>
<td>None</td>
<td>None</td>
<td>Lubricated</td>
</tr>
<tr>
<td>5</td>
<td>CN Profile A</td>
<td>0</td>
<td>None</td>
<td>None</td>
<td>Lubricated</td>
</tr>
<tr>
<td>6</td>
<td>New</td>
<td>0</td>
<td>None</td>
<td>None</td>
<td>Lubricated</td>
</tr>
<tr>
<td>7</td>
<td>New</td>
<td>83.4</td>
<td>-</td>
<td>None</td>
<td>Lubricated</td>
</tr>
<tr>
<td>8</td>
<td>CN Profile A</td>
<td>0</td>
<td>None</td>
<td>None</td>
<td>Lubricated</td>
</tr>
</tbody>
</table>

The L and N hopper car was instrumented by the AAR Technical Center to measure 22 acceleration values, 49 displacement values, wheel/rail forces for one truck set with 1/20 tread profiles, and the train speed. This instrumentation included 14 displacement transducers to measure the relative lateral and angular position of the wheels relative to the rails.

The signals from the transducers were conditioned, digitized and recorded by the instrumentation system on board the AAR 100 instrumentation car. The data was sampled at 100 samples/second and recorded on magnetic tape. These data tapes were subsequently analyzed at the AAR Technical Center and Arizona State University.

The tests were conducted on both tangent and curved sections of track. The tangent test site was a 12,000 foot section of continuous welded rail on the UP mainline in the Mojave Desert between Yermo, California and Las Vegas, Nevada. The curving tests were conducted on Union Pacific track between Sloan and Arden, Nevada.

The tests were carried out by the AAR Technical Center and the UP Railroad. The AAR provided the test car, instrumentation car and test crew while the motive power, caboose and train crew were provided by the Union Pacific Railroad. The tests were under the direction of the AAR Research Center.

Many details of the test vehicle, instrumentation and test conduct are given in [36]. The vehicle description and instrumentation are also discussed at greater length in [30].

A significant portion of the experimental program was devoted to obtaining data for the characterization of the vehicle/roadbed system. This effort included laboratory testing of the test vehicle and its components; on-site measurements of vehicle, rail and roadbed characteristics; and data analysis to obtain the desired system parameters.

Vehicle Characterization

Laboratory tests were conducted by American Steel Foundries (ASF), under the AAR'S Track-Train Dynamics Program to characterize the inertial, geometric and suspension characteristics of a 70 ton Ride Control truck. The data from these tests were used as input to theoretical dynamic analysis of the test vehicle behavior. The tests and test results are discussed in [35].

Martin-Marietta, Denver Division, conducted further tests on a single, completely assembled, 70 ton Ride Control truck [36], to supplement the test data obtained by ASF. These tests were conducted with one of the trucks used in the field test program.

Wheel/Rail Geometry Characterization

One of the most dominant influences on rail vehicle lateral dynamic response is the wheel/rail geometry. Two wheel profiles were used during the field tests. The test vehicle wheelsets with these two profiles, the AAR, 1/20 tapered profile and the Canadian National (CN) profile "A" [37], were measured using a wheel profile measuring machine borrowed from the Deutsche Bundesbahn (DB), the German Federal Railroads.

A rail head profile measuring device also was borrowed from the DB. This device was used to measure transverse rail head profiles at approximately 150 stations in the tangent and curved test sections. Time and manpower limitations precluded measuring more stations, although the information would have been useful in developing power spectral densities for rolling line and other offsets.

The wheel and rail head profiles measured during the field tests were digitized and processed through a modified version of the wheel/rail geometry analysis program described in [7]. Rolling line offset, effective conicity, gravitational stiffness, and other wheel/rail geometric constraint parameters were calculated at each roadbed station for both the AAR and the CN profile wheelsets. Mean values, standard deviations and other statistical measures for these parameters were also calculated for each test section.

Roadbed Characterization

The curved and tangent track geometry was measured by the UP's track geometry car in the Fall of 1975. Due to the long delay before carrying out the field tests, attempts were made during and immediately after the field testing period to resurvey the roadbed with the FRA Track Measurement cars and with the Track Survey Device. The track geometry measurement was measured with FRA Track Measurement Car T1-T3 on November 27, 1977.
As the FRA track measurement car does not provide information about roadbed lateral alignment, a dominant input to the vehicle lateral dynamics, the most useful roadbed geometry data was obtained from the UP’s Plasser-American EM-80 track evaluation car. This data was processed by the spectral analysis program described earlier.

The roadbed centerline alignment PSD for the 12,000 foot tangent test section is shown in Figure 4-2. The two large peaks in this PSD are introduced as a consequence of compensating for the chordal offset measurement process used by the EM-80.

Test Conduct

The field tests themselves were conducted between November 1976 and January 1977. Four different types of tests were made with each of the 8 configurations: curving tests, random response tests, forced response tests, and creep tests.

Curving Tests

Two curves, 1 degree and 6 degrees, were traversed at 3 speeds. The speeds were intended to be below, near and above balance speed.

Our intention was to evaluate the theory for steady state rail vehicle curving by comparison of predicted and measured wheelset lateral and yaw displacements for the various vehicle configurations and test conditions. Comparison with the steady state wheel/rail force data was to be made, where possible. However, as explained earlier, the wheel/rail displacement and wheel/rail force data were not adequately calibrated, and could not be used for these comparisons.

Random Response Tests

These tests were conducted at different speeds on the tangent mainline track. The primary disturbance acting on the vehicle in these tests was the random irregularity of the roadbed. Data from these tests was used for comparison with random response and stability analyses.

Forced Response Tests

It was decided early in the test planning that it would be desirable to evaluate the vehicle response to a known, deterministic input. A study was conducted to evaluate roadbed and onboard inputs, and to find the most effective form for each. An onboard system was selected.

The test vehicle was equipped with a hydraulic truck forcing system that exerted a torque between truck and car body. This system, when pressurized, caused an initial translation and angular displacement of the truck components, providing a controllable initial condition. When the forcer pressure was released, the subsequent transient response was observed to extract frequency and damping information.

The forced response tests were also conducted at several speeds for each configuration on the same tangent test section as the random response tests. The test results were eventually used for comparison with theoretical stability analyses.

Creep Tests

Pre-test sensitivity studies indicated a strong influence of the creep coefficients on the vehicle behavior. For this reason, a fourth series of tests was planned and executed in an attempt to determine the creep coefficients more directly. These tests, run at slow speeds of 5-10 mph, involved applying measurable torques between the truck and car body and recording the resulting displacement and angular positions of the wheelsets relative to the rails. The torques were applied with the hydraulic forcer system described earlier.

The creep coefficients were to be determined by an identification procedure utilizing the equilibrium equations for the vehicle and the measured wheelset and car component displacements. Unfortunately, the data from these creep tests also cannot be used because the initial conditions for the wheel/rail transducers were not obtained. This prevents extracting of the steady state lateral and yaw displacements that are needed to determine the creep forces.

Despite the failure of these creep tests, we believe that field testing to establish the creep force conditions should be a part of all future tests.

RESULTS

Test data was furnished to Arizona State and Clemson Universities in the form of digital magnetic tapes. This data was analyzed with the tools described earlier, and compared with theoretical results. For reasons discussed earlier, only tangent test data was used.

Test Results

Displacement transducer data proved to be more useful for our purposes than acceleration data. Initially, the raw displacement data was combined into "model" variables and the results stored on disc and tape. All variables to be analyzed were plotted as functions of time. Portions of data from each configuration and speed of interest were incorporated for analysis. Figure 4-2 shows a portion of the Configuration 2, A-truck lateral displacement during an unforced tangent track run at 35 mph. We found it essential to study plots of all data to be analyzed before proceeding. For
example, in Figure 4-3, periods of hunting followed by periods of non-hunting are seen. Because the term damping ratio cannot be applied to hunting behavior, a fundamentally nonlinear phenomenon, it is only meaningful to estimate damping ratios for periods without hunting.

Selection of Signals

In theory, all modes of vibration are present in every motion of a coupled dynamic system. From that viewpoint, any of the model variables could be used for the frequency and damping ratio estimates. However, the least damped mode is dominant in some signals. We found that the truck lateral displacement signals were best for estimating parameters of the least damped mode, although this mode is readily observable in the truck yaw, truck warp, and wheel displacement signals.

Random Response

Test vehicle response to random rail irregularities was studied using the spectral analysis computer program. A typical PSD plot is shown in Figure 4-4. The least damped mode is readily seen in this plot, the large peak at about 1.4 Hz. The smaller peak at 8 Hz is associated with the wheel revolution rate. The peaks at 20 and 40 Hz are noise from the motor-generator set on the data collection car.

The PSD plots were used, in turn, to estimate damping ratios and frequencies for the least damped modes of motion, using the method described earlier.

Test vehicle response to random rail irregularities was also studied using the Random Decrement Technique. A typical Random Decrement Signature and the least squares curve fit to it are shown in Figure 4-5. Damping ratios and frequencies are directly available in the parameters of the curve fit to the signature.

Both the PSD and Random Decrement techniques provided essentially the same results for frequency and damping ratio. The techniques should be viewed as two different methods of extracting the same information from a signal. Both take about the same amount of computer time, in our implementations. The advantages of the Random Decrement are that it works for all values of the damping ratio, however large or small, and that it makes the frequency and damping ratio estimates automatically.

The primary disadvantage to the Random Decrement Technique is that a judgment is required as
to how much of the signature to fit with the decaying sine wave. The signature degenerates at low amplitude values, so it is always truncated at some point. We believe that this degeneration may be due to nonlinearities in the test vehicle.

The end result of the damping and frequency estimation process is the frequency and damping ratio vs. speed characteristic. Figure 4-6 shows the experimentally determined natural frequency and damping ratio as functions of speed for Configuration 4.

**Forced Response Analysis**

Test vehicle response following application of the truck forceors was studied using the logarithmic decrement technique. Figure 4-7 shows a typical time history of the A-truck lateral displacement during a forced run.

Damping ratios were computed from signals such as that of Figure 4-7. Figure 4-8 illustrates the damping ratio vs. speed information obtained from such an analysis for Configuration 5. Interestingly, these values are about one order of magnitude less than the corresponding values found by analysis of the random response data.

The difference between the random and forced response results is probably due to nonlinearities in the vehicle suspension. In a nonlinear system such as the test vehicle, where Coulomb friction is the dominant nonlinearity, the effective damping decreases with increased amplitudes of motion. Because the forced runs began with the flanges hard against the rails during the empty vehicle runs, the amplitudes were at maximum values. In contrast, during the unforced runs the amplitudes were smaller. As a result, smaller estimates of damping would be expected in the forced response situation.

**Vehicle Hunting Characteristics**

An important result of the field tests is a characterization of the hunting performance of the test vehicle in its eight configurations. Discussions of hunting behavior are often conducted in terms of "the hunting speed" of a vehicle as though hunting speed were a unique property of a vehicle. These tests demonstrated that, in reality, the hunting behavior of a freight vehicle can not be characterized by a single number with the dimension of speed. There is actually a transition speed range in which intermittent hunting occurs. Below this range hunting never occurs, and above it sustained hunting always occurs. A typical burst of intermittent hunting, this one of about 30-seconds duration, is shown in Figure 4-3. This data was recorded on a constant 35-mph test run of Configuration 2. It should be noted that this type of hunting can easily go undetected by a test crew.

Figure 4-9 summarizes the hunting characteristics of the eight configurations. The ordinate on these bar-type charts is percent of time the vehicle was hunting at a given speed. The data points used in constructing the graphs are shown. In every case a point is shown for zero-percent hunting. This point represents the speed at and below which no hunting was observed to occur. A point is also shown, when available, for 100-percent hunting, which is the speed at and above which sustained hunting was always observed to occur.
CONCLUSIONS

Field Test Conduct

We have reached several observations and conclusions concerning the test planning and conduct that we believe may be useful to others involved in similar tests. Concerning the vehicle, a very thorough effort was made to characterize the test vehicle. However, only one truck was characterized, and that effort was carried out in Denver many months prior to the test. Consequently, differences in suspension stiffness and friction between the two trucks, and the variations in suspension friction with environmental conditions remain undetermined. It would be desirable to identify these characteristics at the test site.

The vehicle instrumentation and data recording system used in these tests were well thought out, and they performed well during the tests. The wheel/rail displacement probes, in particular, provided reliable and useful data. The greatest shortcoming of the tests, however, was the failure to obtain initial conditions for these devices. A simple procedure to zero these probes should have been developed.

We would have preferred, given the limitations of testing resources (and the benefit of hindsight), to have more test data for fewer vehicle configurations. Longer runs at the same speed, and runs at more speeds would have given enough data to analyze the anomalous behavior that often occurred. Holding speed more nearly constant during the test runs also would have made the data more useful.

One of the shortcomings of our tests was the lack of adequate roadbed geometry characterization. As explained earlier, alignment, profile and gauge data was only gathered one year before and after the tests. In addition, there is strong evidence that the alignment data recorded were in error. The actual roadbed condition during the tests was not known, nor were we able to synchronize the vehicle test data with roadbed measurements. This proved to be particularly unfortunate, because we found that nonlinear friction in the vehicle suspension causes the vehicle behavior to depend strongly on the roadbed disturbances.

Test Results

No unique hunting speed exists for a given vehicle configuration. Instead, a hunting speed range exists. In the lower portion of this speed range hunting may be intermittent. Above some speed in this range sustained hunting always occurs.

The kinematic mode of the test vehicle dominated the response of the vehicle. It is fairly well recognized by now that the kinematic mode of a vehicle exhibits a frequency that is nearly directly proportional to speed. This was found to be the case in our tests. The PSD analysis clearly shows the majority of energy to be associated with the kinematic mode. This mode is so strong that it can be observed directly in the time response of the vehicle.

In summary, we regard these tests as one of a series of tests to obtain data for comparison with theoretical vehicle dynamic analyses. We believe that a great deal of useful information was obtained in this project, and that better information will be obtained in future tests by building on the foundation of information obtained in these tests.
Chapter 5

COMPARISON OF TEST AND THEORY

INTRODUCTION

An overview of our efforts to compare and obtain agreement between test and theoretical results for the lateral dynamic behavior of a rail freight car is presented in this chapter. This work is discussed fully in [30].

The process reported in [30] and summarized here is only a first step toward validation of rail vehicle dynamics theory. Numerous ideas that might lead to better correlation were generated but could not be pursued within the time and effort constraints of the project.

Experimental data were compared with results from the following three analysis methods: linear eigenvalue-eigenvector analysis, quasi-linear statistical linearization analysis, and direct analog integration using a hybrid computer. The following discussion is organized around these three analytical approaches.

LINEAR EIGENVALUE ANALYSIS

A linear eigenvalue analysis of the 9 degree-of-freedom freight car model was used to develop theoretical estimates of the damping ratio and frequency of the least damped or hunting mode. These estimates were made at a series of speeds and results were compared with those obtained by random decrement and spectral analysis of test data.

Sinusoidal input describing functions were used to obtain estimates of linear equivalent values for the suspension and wheel/rail geometry nonlinearities. Theoretical results were obtained for Configuration 6 for a range of creep coefficient values. Three cases were considered in calculating the equivalent linear parameters using sinusoidal input describing functions: (a) nominal amplitudes and frequency, (b) small amplitudes and low frequency, and (c) small amplitudes, low frequency, and high conicity. Results for the speed dependency of frequency and damping ratio of the hunting mode are shown in Figure 5-1.

As may be seen, there is a strong dependency of the theoretical results on the values of the creep coefficients (one of the unknown parameters of the system). None of the cases in Figure 5-1 show good agreement between theory and test. After evaluating the possible reasons for these discrepancies [30], we have tentatively concluded that it is nearly impossible to represent accurately actual freight car behavior with a purely linear model due to the strong nonlinearities such as Coulomb friction that are present in this vehicle.

STATISTICAL LINEARIZATION ANALYSIS

The statistical linearization approach was used to compute PSD's of vehicle response variables (front and rear truck absolute and relative lateral displacement) for several configurations of the nonlinear vehicle. These were compared with results obtained from the field tests.

Figure 5-1a Damping Ratio and Frequency for Nominal Describing Function Values

Figure 5-1b Damping Ratio and Frequency for Small Amplitude Describing Function Values

Figure 5-1c Damping Ratio and Frequency for Small Amplitude Describing Functions and Wheel Conicity of 0.075.
Sample results where good agreement between analysis and test was obtained are shown in Figures 5-2 and 5-3. Other theoretical results obtained did not show as good agreement with test.

While substantial effort was expended in obtaining results and making comparisons, we did not have the time or funds to pursue fully questions that arose during the investigation or to examine the full range of configurations, speeds, and response variables for which test data were available. Nevertheless, the results obtained [30] indicate the potential value of the statistical linearization approach.

HYBRID COMPUTER ANALYSIS

A simulation of a 5 degree-of-freedom half-car freight car model was conducted using the hybrid computer facility of the Clemson Engineering Computer Laboratory [22,30]. Actual nonlinear wheel/rail geometric constraint functions for small contact angles were used as were simplified nonlinear suspension characteristics determined by tests [36]. Excitation due to lateral track alignment irregularities was simulated and PSD’s for the response variables were calculated. These were then analyzed to obtain estimates of the damping and frequency of the hunting mode. Additionally, limit cycle studies were conducted to determine the sensitivity of the nonlinear hunting mode response to variations in creep coefficients, suspension, and loading characteristics. The response to the truck forces used in the field tests was also simulated and the damping and frequency of the hunting mode were calculated.

An example of the simulated time history of the response of Configuration 6 to both centerline alignment irregularities and the truck forcer is shown in Figure 5-4. Spectra calculated for truck relative lateral displacement for Configuration 6 are shown in Figure 5-5. These spectra compare favorably on a qualitative basis with those obtained from field test data.

One of the main purposes of the hybrid computer analysis was to apply the data analysis methods used with the field test data to the response data obtained from the hybrid simulation, and to evaluate the utility of the different methods. On the basis of the hybrid computer studies we concluded that, for the highly nonlinear freight car, the utility of the damping ratio as an indicator of stability margin and the likelihood of a hunting limit cycle at a given speed is open to question.

[30].

Figure 5-2 Comparison of Analysis and Test Results for Configuration 6 at 25 mph (Creep Coefficients = 75% Kalker)

Figure 5-3 Comparison of Analysis and Test Results for Configuration 6 at 25 mph (Creep Coefficients = 75% Kalker)
SUMMARY

Limited efforts (due to time and budgetary constraints) were made to compare test results with theory. Based on these efforts, we have concluded that it is nearly impossible to use purely linear models to represent accurately the lateral dynamic behavior of the highly nonlinear North American freight car.

Both good and poor agreement of theory and test were obtained with the statistical linearization approach. As several critical parameters (creep coefficients etc.) and the inputs (cross-level, alignment, wheel-rail offset terms) were unknown, we cannot draw firm conclusions regarding the agreement. However, it appears that the statistical linearization approach has considerable promise and efforts should be made to apply and refine it further.

The results of the hybrid simulation of the half car model demonstrate that damping ratio of the hunting mode is not a good indicator of either stability margin or the possibility of a hunting limit cycle for this highly nonlinear freight car [30]. However, the agreement between the hybrid and experimental results indicates that the nonlinear model used here is probably a valid representation for freight car dynamics.
During the course of this project, a series of theoretical models, associated solution methods, and computer programs for predicting freight car lateral dynamic response were developed. Linear, quasi-linear, and nonlinear analyses of these models for freight car behavior on tangent track were implemented. A comprehensive field test program was planned and conducted for the express purpose of providing test data for comparison with theory. This test program included characterization of the vehicle and roadbed, extensive instrumentation of the vehicle, and operation of the test vehicle in several configurations and modes. The data from these tests was reduced and analyzed by several different methods, some of which had not been used in rail vehicle testing prior to this project. Due to time and funding constraints, only limited comparisons of theory with the test results were conducted.

Six formal project reports were prepared. These are listed together with the documentation pages in Appendix A. Seven computer program users' manuals were written describing programs that were developed and made available to the public through the National Technical Information Service (NTIS). Six of these are listed together with documentation pages in Appendix B. The seventh is contained within the report, "Analytical and Experimental Determinations of Nonlinear Wheel/Rail Geometric Constraints" described in Appendix A. Ten informal reports were prepared as needed to document results or to convey information to interested parties. These are listed in Appendix C. Eleven papers or presentations at national meetings were made based wholly or in part on this project.

MODELING AND ANALYSES

Our approach in this project has been to develop and investigate the conditions of applicability of a number of modeling and analysis approaches. The rationale behind this multi-faceted approach is to provide a variety of model and analysis techniques that can be tailored to the particular analytical objective.

Linear and nonlinear analyses both have a place. We found that the linear analysis approaches cannot be used to predict accurately the dynamic response of the highly nonlinear freight car. However, they can be used as inexpensive "stepping-stones" to the nonlinear analysis and, when used with care and judgment, can help in obtaining a qualitative understanding of the response. The quasi-linear and hybrid computer analyses of the nonlinear freight car yielded results that appear to be more representative of the actual behavior encountered.

Simpler vehicle models appear to be useful in many situations. Our analysis of field test data indicates that the 1 degree-of-freedom model of the freight truck that is incorporated in the 9 degree-of-freedom vehicle model is appropriate for describing the conventional freight truck.

If the car body is sufficiently rigid such that the natural frequencies of the first lateral bending mode and the first torsional mode are significantly higher than the rigid body and hunting modes, the 9-degree-of-freedom model should be a reasonable representation for the lateral dynamics of a conventional freight car. For car bodies that are flexible, the 23 degree-of-freedom vehicle model that incorporates a 5 degree-of-freedom car body may be used. If it is desired to model trucks with lateral and longitudinal primary suspension elements, direct wheelset interconnections, and independently rotating wheels, one may choose from the 17, 19, or 23 degree of freedom models (Table 2-1). Quasi-linear and hybrid computer analyses of these models were not developed, although these would be quite useful for investigation of advanced truck and car design concepts.

FIELD TESTING AND COMPARISON WITH THEORY

The field tests conducted in concert with this project yielded much useful information for comparison with theory. Time and budgetary constraints precluded our making use of all the data or carrying out as complete a comparison as possible. An oversight in the conduct of the field tests rendered unusable the data obtained from the creep and curving tests. The comparisons we did make indicated that purely linear models cannot accurately represent the lateral dynamic behavior of the highly nonlinear North American freight car. As a corollary, the data analysis techniques used to determine modal damping and frequency gave highly variable results. These approaches, including the random decrement method, log decrement analysis and analysis of PSD's, imply an assumption of nearly linear behavior when used to estimate damping ratios. We have concluded that damping ratio is not a good indicator of stability margin for the nonlinear freight car.

The comparisons of test results with results from the statistical linearization (or quasi-linear) analysis provided mixed returns. Because several unknown parameters had to be guessed or neglected (creep coefficients, alignment and cross-level inputs, rolling line offset, etc.) no firm conclusions regarding the validity of the model and analysis approach can be drawn. However, based on the results obtained, the statistical linearization approach (while more difficult to use than the linear analyses) appears to have considerable promise.

FUTURE WORK

As in any project of this scope and duration, many subjects and questions could not be investigated fully. The following open questions should be topics for future work:

1. Analysis of the field test data and comparison with theoretical results for the vehicle configurations that were not studied in this project.
2. Further development of the existing statistical linearization approach and application to configurations and conditions that were not studied in this project.
3. Refinement of the statistical linearization algorithm to include cross level and rolling line offset inputs and extension to more complex vehicle models.
5. Completion of the development of a nonlinear dynamic model and analysis for investigation of curve entry and negotiation of freight vehicles.

6. Development of identification and simple test techniques for determination of creep force characteristics and suspension parameters.
REFERENCES


27. Anon., "Freight Car Truck Design Optimization:


Appendix A

FORMAL PROJECT REPORTS

The following formal reports have been prepared during this project:


5. Freight Car Dynamics: Field Test Results and Comparisons with Theory, June 1981.


Documentation pages for each report follow. The reports are available through the National Technical Information Service (NTIS).
**Analytical and Experimental Determination of Wheel-Rail Constraint Relationships**

**Abstract**

Wheel/rail geometric constraint relationships, such as the effective conicity and gravitational stiffness, strongly influence the lateral dynamics of railway vehicles. In general, these geometric constraints are nonlinear functions of the wheelset lateral displacement. This report describes the development and validation of an analytical procedure to determine these nonlinear functions for arbitrary wheel and rail profiles. Data for validation of this analysis was obtained experimentally. Experimental and analytical data for three validation cases is presented. Results of a limited parametric study are also reported. The computer program for the analytical procedure is described and documented.

**Key Words**

- wheel/rail kinematics
- wheel/rail geometry
- railroads
- conicity, gravitational stiffness
- wheelset
- wheel profile, rail profile

**Security Classif. (of this report)**

UNCLASSIFIED
This report presents the development of general analytical models for use in exploring the nature of freight car hunting and for finding means of controlling the hunting behavior.

These models result from one aspect of the "Freight Car Dynamics" research project conducted by Clemson and Arizona State Universities in cooperation with the Association of American Railroads. This effort is directed at developing models to describe rail vehicle curving behavior and response to track irregularities in addition to those models presented in this report that deal with hunting.

First, a model of a wheelset with lateral, yaw, and axle torsional degrees of freedom is developed. Secondly, two such wheelsets are included in a general model of a 9 degree of freedom truck that has lateral, yaw, and warp degrees of freedom in addition to relative lateral and yaw motions of the wheelsets with respect to the truck frame. By suitable choices of primary suspension elements, this general model may be specialized to become (1) a roller-bearing freight truck, (2) a plain-bearing freight truck, (3) a roller-bearing truck with primary suspension elements, (4) a passenger truck, (5) a generic model of a freight truck with interconnected wheelsets, or (6) a rigid truck. Finally, two such truck models are combined with a car body that has lateral, yaw, and roll rigid body degrees of freedom plus two degrees of freedom that serve to approximate the first lateral bending and torsional modes. For all three models, the effects of design parameters on the critical speed for hunting are examined.
Coulomb friction, such as found in the suspension system of railway freight cars, can strongly influence dynamic behavior. The coulomb friction nonlinearity must be accurately implemented in computer simulations of multi-degree-of-freedom dynamic models. This report proposes three computer models for friction and analyzes their performance in analog and digital computer simulations. Simulation techniques used are described in detail. Performance of each friction model is compared to analytical results. The accuracy, advantages, and disadvantages of each model are discussed. The report concludes with recommendations on the use of the proposed friction models.
**Abstract**

A linear model of the vertical dynamics of a railcar was validated by the application of spectral techniques to experimental data. Track input spectra were computed from test track surface measurements gathered in the TDOP test program. Acceleration measurements of a freight car were used to compute vehicle acceleration spectra in response to the test track. The corresponding response of the linear model was computed from the analytical transfer functions and experimental track input spectra. Validation of the linear model was based upon a comparison of corresponding analytical and experimental vehicle acceleration spectra.

The truck suspension was isolated and analyzed from experimental measurements of corresponding truck and car body accelerations. Spectral functions were employed to evaluate the assumptions of suspension linearity.
Field tests of a conventional rail freight car were conducted to provide data for comparison with theoretical analyses of rail freight car dynamic behavior. These tests, carried out by the Association of American Railroads and the Union Pacific Railroad, were designed to provide experimental information concerning modal damping, modal frequencies, mode shapes, motion amplitudes, critical hunting speeds, wheel-rail forces, and creep coefficients for comparison with theoretical results.

Alternative techniques for theoretical freight car analysis, data processing, and comparison of theory and experiment are discussed. The 80-ton open-hopper test car parameters, the wheel-rail and roadbed geometry, the test conduct and the data analysis procedures are described. Field test results in the form of time series data, RMS values, modal frequency and damping characteristics, and spectral analysis results are presented and discussed. Comparisons of test results with theoretical results of linear eigenvalue analyses, quasi-linear random response analyses, and hybrid computer simulations are given.
**Abstract.**

The objective of this research project was to develop techniques to analyze the lateral dynamic behavior of railroad freight cars. The effort included development and correlation of theoretical techniques for predicting freight car dynamic behavior, and use of the techniques to investigate the behavior of present and proposed designs. The project was sponsored by FRA with support and cooperation from the Association of American Railroads and the Union Pacific Railroad.

A number of models and analysis approaches were developed for freight car behavior on tangent track. These models differ widely in complexity. An output of the project is the understanding of the appropriate use of each model and analysis technique.

Extensive field tests were planned and carried out with the cooperation and support of the Association of American Railroads (AAR), the Union Pacific Railroad (UP), and Martin Denver Division. These tests were conducted by the AAR and UP. Eight different vehicle configurations were tested, each at several speeds on both tangent and curved track. Data obtained from these tests were used for comparisons with theoretical predictions of vehicle response.

**16. Key Words.**
freight car, rail vehicle, dynamics, rail vehicle testing, rail vehicle modeling

**18. Distribution Statement.**
Document is available to the public through the National Technical Information Service. Springfield, VA 22161
Appendix B

COMPUTER PROGRAM USERS' MANUALS

The following computer programs were developed, documented and made available to the general public during this project:


Documentation pages for these Users' Manuals also follow. Both the manuals and the computer programs are available from NTIS.

*A users' manual for the symmetric wheel/rail contact characterization program is contained in, "Analytical and Experimental Determination of Nonlinear Wheel/Rail Geometric Constraints," December 1975.
16. Abstract

Wheel/rail geometric constraint relationships, such as the effective conicity and gravitational stiffness, strongly influence the lateral dynamics of railway vehicles. The principal curvatures of wheel and rail profiles are important parameters in the determination of creep coefficients used in rail vehicle models. In general, these geometric constraints and profile curvatures are nonlinear functions of the wheelset lateral displacement. This report is a users' manual for a computer program written in FORTRAN IV that uses iterative procedures to determine these nonlinear functions for arbitrary wheel and rail profiles. The program computes the wheel/rail contact positions, geometric constraint functions, and profile curvatures for any given wheel profile, rail profile, rail cant angle, and rail gauge for an asymmetric wheelset on asymmetric rails. Analytical methods used and program input and output are described. Results are in the form of printout, punched cards and drum plotter plots. The users' manual includes program listings, sample deck set-ups, and sample run output.
The conversion of the computer program, "Simplified Theory of Rolling Contact," (used for calculation of a nonlinear creep force-creepage relationship) from the original Algol language to Fortran is considered. The Algol program was written by Professor J. J. Kalker and was derived from the paper, "Simplified Theory of Rolling Contact," Delft Progr. Rep., Series C: Mechanical and Aeronautical Engineering and Shipbuilding, 1 (1973), pp. 1-10. A significant number of changes was made in the program for more convenient use; however, the fundamental equations remain unchanged. The results were checked in detail to insure agreement with the original solution.

The program gives an appropriate solution for the resultant tangential creep forces and spin moment acting between two bodies of equal linearly elastic material properties. The creep forces and spin moment are due to lateral, longitudinal, and spin creepages. Assumptions corresponding to the Hertz contact theory are implied and two additional simplifying assumptions are made, resulting in a significant reduction in computation time as contrasted with previous solutions.

Two separate computer codes were developed, the first being the general solution with extended input and output, and the second a shortened version primarily intended for use as a subroutine. Surprisingly good agreement is found to exist between the "Simplified Theory" and published experimental results for a wide range of contact ellipse eccentricity.
The conversion of the computer program, "A Programme for Three-Dimensional Steady State Rolling" developed by Professor J. J. Kalker, from the original Algol language to Fortran is considered. This program determines the resultant creep forces and moment for steady state rolling of two bodies of equal or unequal linearly elastic material properties.

A related manual for Kalker's "Simplified Theory of Rolling Contact" is considered in the report "User's Manual for Kalker's Simplified Nonlinear Creep Theory," by James G. Goree and E. Harry Law, FRA/ORD-78/06 Contract DOT-OS-40018, December, 1977. The program considered in the present report concerns the same problem except for the extension to unequal materials. It is found that, for equal materials, the "Simplified Theory" gives approximately the same results as the exact solution in most cases and in those instances where some difference was noted, the simplified theory appears to be in better agreement with experimental results. In addition, the simplified theory reduces the computation time by a factor of approximately 50 to 100.
A program written in FORTRAN IV is described that uses the Hertz theory of rolling contact between two bodies and Kalker's linearized theory of creep to determine the geometry of the contact patch between railway vehicle wheel and rail and the creep coefficients that characterize the linearized creep forces between wheel and rail. The program input, output and the subroutines used are described herein and the results are in the form of printout. The manual includes program listings, sample deck set ups and sample run outputs. Two other user's manuals for determination of creep forces and moments have been issued under this contract. These are "Users' Manual for Kalker's Simplified Nonlinear Creep Theory" by J.G. Goree and E.H. Law and "Users' Manual for Kalker's Exact Nonlinear Creep Theory" by J.G. Goree.
The dynamic performance of a rail vehicle is characterized by its curving behavior, vibration response and stability. The stability of a linear system can be studied by such classical methods as eigenvalue/eigenvector analysis. This report is a Users' Manual for four programs written in FORTRAN IV that use the eigenvalue/eigenvector analysis to determine the lateral stability of the 9, 17, 19, and 23 degree of freedom linearized models of the North American Freight Car. The program input, output, and the subroutines used are described herein and the results are in the form of printout. The manual includes program listings, sample deck setups and sample run outputs.
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15. Key Words | Rail vehicle dynamics, forced lateral response, rail freight car.
16. Abstract | This manual documents a FORTRAN IV computer program that solves for the forced lateral sinusoidal and random response of a linear, 9 degree-of freedom freight car. The vehicle model represents the lateral dynamics of typical North American freight cars equipped with three piece trucks. Responses to both roadbed centerline alignment and cross level irregularities are computed.

The response is calculated using frequency domain techniques. The steady state sinusoidal transfer functions are computed, response power spectral densities obtained and R.M.S. values found by integration of the power spectra. Plots of selected output power spectra are prepared.

The manual briefly describes the vehicle and roadbed model and the solution technique. The program description, a sample run and a complete program listing are included.

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Appendix C

INFORMAL REPORTS

In addition to the formal documents listed in Appendices A and B, the following informal reports were prepared as needed to document results or to convey information to interested parties:


