Parametric Studies on Lateral Stability of Welded Rail Track

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### Abstract

Thermal buckling of railroad tracks in the lateral plane is an important problem in the design and maintenance of continuous welded rails (CWR). The severity of the problem is manifested through the increasing number of derailments which are attributable to track buckling, indicating a need for developing better control on the allowable safe temperature increase for CWR track.

The work reported here is part of a major investigation conducted by the Transportation Systems Center (TSC) for the Federal Railroad Administration (FRA), on the analytical predictions of critical buckling loads and temperatures, and deals with a parametric investigation of the buckling response of CWR track. Buckling temperature and safe allowable temperature increase values are predicted for both tangent and curved tracks, as influenced by several key parameters, including track lateral and longitudinal resistances, lateral misalignments and rail size. Results of sensitivity analyses over a practical range of the parameters are presented and a suitable design criterion to ensure stability of CWR is outlined.

### Key Words

- Track Buckling
- Track Lateral Stability
- Buckling Tests
- Buckling Analysis
- Continuous Welded Rail (CWR)
# METRIC CONVERSION FACTORS

## Approximate Conversions to Metric Measures

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## Approximate Conversions from Metric Measures

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## Notes
- 1 in. = 2.54 cm (exactly).
- For other exact conversions and more detailed tables see Table 588, Misc. Publ. 286. Units of Weight and Measures. Price $2.25 IBM Catalog No. C1310286.
PREFACE

Under the Federal Railroad Administration’s (FRA) Improved Track Structures Research Program, the Transportation Systems Center (TSC) is conducting research to develop the engineering basis for more effective track safety guidelines and specifications. The intent of these specifications is to ensure safe train operations while allowing the industry increased flexibility for cost-effective track engineering and maintenance practices.

One of the major safety issues currently under investigation under this program deals with track buckling. The work reported here is part of this investigation and deals with a parametric investigation of the buckling response of CWR track. Analytic predictions and sensitivity assessments over the practical range of critical parameters are presented, and a suitable design criterion to ensure the buckling safety of CWR is outlined.
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LIST OF SYMBOLS AND ABBREVIATIONS

x coordinate axis in the longitudinal direction
E Young's Modulus for rail steel
A rail cross-sectional area
I rail area moment of inertia about the vertical axis
ΔT rail temperature (above the stress-free temperature)
ΔTs safe temperature (above the stress-free temperature)
ΔTb buckling temperature (above the stress-free temperature)
P rail compressive force in the buckled zone
w compressive force in the rails
u lateral deflection
w' axial displacement in the adjoining zone
 primes denote derivatives with respect to x
α coefficient of thermal expansion
Fo constant lateral resistance
f₀ constant longitudinal resistance
τ₀ constant torsional resistance
2L test track length
2L buckling length
2Lo length of misalignment
δ₀ misalignment amplitude
k end stiffness
R radius of curvature
μ₁ stiffness parameter for lateral resistance
μ₂ stiffness parameter for longitudinal resistance
μ₃ stiffness parameter for torsional resistance
2Lₙ breathing length
2Lₚ length of track experiencing prebuckling displacements due to finite stiffness at the ends

ix/x
SUMMARY

The increased utilization of continuous welded rail (CWR) in U.S. tracks has resulted in an increasing number of accidents attributable to derailments induced by thermal buckling of railroad tracks. In an effort to improve the safety of CWR, experimental and analytic investigations are being conducted by the Transportation Systems Center (TSC) supporting the safety mission of the Federal Railroad Administration (FRA). This report describes a part of these investigations dealing with the parametric study of buckling response of CWR, and presents the results applicable for improved safety, design and maintenance practices.

Theoretical analyses verified earlier against field test results are utilized to conduct the parametric investigation of buckling stability of CWR track. Buckling temperature and safe temperature increase values are predicted for both tangent and curved tracks as influenced by several key parameters, including track lateral and longitudinal resistances, lateral misalignments and rail size. Results of sensitivity analyses over the practical range of parameters are presented and a design criterion for buckling safety of CWR based on the safe temperature increase concept is also outlined.

Predictions on the influence of track curvature, initial misalignments, and track resistance parameters on track stability indicate that curved tracks exhibit significantly lower buckling strengths than tangent tracks, and that misalignments and track lateral resistance have a significant effect on the lateral buckling response.

Predictions also indicate that buckling of short test tracks (or long tracks heated over finite lengths) are significantly influenced by the end conditions and the length of the heated track. This is of paramount importance in the proper interpretation of buckling tests results and in the design of buckling tests.
1. INTRODUCTION

Thermal buckling of tracks in the lateral plane is an important consideration in safe operations of CWR. The Federal Railroad Administration, through the Transportation Systems Center, has initiated a major research program in this area. The program consists of both theoretical investigations of the mechanics of buckling and field tests on typical mainline tracks. The research is expected to lead to a set of safety specifications and guidelines for CWR installation for the U.S. railroad industry, which is experiencing an increased number of derailments attributable to track buckling.

In recent years, significant progress has been made on the subject both in theoretical and experimental directions. Kerr [1]* presented a post-buckling analysis for lateral buckling of tangent tracks without imperfections. Samavedam [2] presented a complete static buckling theory for both tangent and curved tracks taking into account lateral misalignments, nonlinearities in the resistances, and lateral loads. Samavedam [3] also conducted buckling tests on a specially built track at Old Dalby, England. The experimental findings were in reasonable agreement with the theoretical predictions. Further contributions to track buckling were presented in a recent work by Kish, Samavedam and Jeong [4] which deals with the buckling problem of finite tracks and analyzes all the recent buckling tests conducted in the U.S.

On the basis of these recent investigations, it was concluded that the theory for the static buckling of CWR is in a well developed state to justify a numerical parametric study of the problem. Such a study will help reduce the number of experiments which are very expensive, involved, and operationally often prohibitive. The work reported herein gives the results of this parametric study, which had the following major objectives:

- Identify critical parameters governing the lateral stability of CWR on the basis of previous theoretical considerations.

*[ ] denotes references.
Perform sensitivity studies over a practical range of the parameters and suggest practical means of controlling the parameters.

Outline a suitable design criterion to ensure the stability of CWR track.

The report is organized such that those not interested may skip Section 2 dealing with theoretical considerations and follow the remaining sections which deal with the practical aspects of the lateral buckling of CWR. Theoretical researchers may supplement Section 2 with reading of the previous reports by Kerr [1], Samavedam [2] and Kish, et al., [4] for a more complete treatment of the lateral buckling theory.
2. THEORETICAL CONSIDERATIONS

Previous works by Kerr [1], Samavedam [2], Kish [4], et al., describe the general theory of lateral buckling of continuously welded rails. There are basically three important temperatures which influence the lateral response of tracks, namely:

- Neutral Temperature
- Safe Temperature
- Buckling Temperature

At the neutral temperature, the longitudinal force resultant in the rails is expected to be zero. This can be considered as the reference temperature to which the safe and the buckling temperature increases and corresponding force levels are referenced.

The fundamentals of track buckling can be explained by means of the equilibrium curve (Figure 1), which represents the relationship between the temperature increase above its neutral temperature and maximum track lateral displacement. There are two "ascending" branches, OB and SC, which represent stable equilibrium configurations whereas the descending branch BS represents unstable equilibrium positions. There are two critical points on the curve denoted by $\Delta T_B$ and $\Delta T_S$. The former is called the buckling temperature increase, which is the temperature increases above neutral at which the track will explosively displace into the next stable branch SC. The temperature $\Delta T_S$ is called the safe temperature increase. Below this temperature, the track has only one stable equilibrium branch, OS', hence the track can be exposed to ambient heating without the risk of buckling, provided that there are no external vehicle induced loads and dynamic effects in the track. It is to be emphasized that Figure 1 shows all possible positions of equilibrium which are analytically predicted. In actual practice or in a buckling test, the temperature increase versus deflection response is that of OBC. If, for example, the temperature increase does not quite reach B and the temperature is decreased to S', it is obvious that the deflection response will be back down on BS' (and not BS) assuming elastic prebuckling displacements. At $\Delta T_B$ buckling takes place, and although there is axial force loss associated with buckling,
FIG. 1 - TEMPERATURE-DEFLECTION EQUILIBRIUM CURVE FOR TRACK LATERAL BUCKLING
there is no temperature loss, i.e. the buckling response is that of BC and not BSC. If after buckling the temperature continues to increase, the response is that of CD. For details of the other relevant stability aspects, see [1,2,4].

Design of CWR track is based on the three temperatures discussed above. These temperatures are dependent on several independent variables of tracks, namely:

- Resistance Characteristics
- Imperfections in Lateral Alignment
- Curvature
- Rail and Fastener Properties
- Track Length and End Stiffness (in Test Tracks)

These parameters are briefly discussed below.

2.1 RESISTANCE PARAMETERS

The restraint offered by the ballast to the track superstructure (ties, fasteners, and rails) can be described by the following parameters:

- Lateral Resistance
- Longitudinal Resistance
- Torsional Resistance

**Lateral Resistance**

This is the resistance offered by the ballast as the ties tend to move laterally with the rails. The relationship between the resistance and the tie displacement is nonlinear as shown in Figure 2. For small displacements it is monotonically increasing and at some "yield value" it tends to level off. The resistance can be represented by some function of the form

\[ F(w) = F_0 \tanh(\mu_1 w) \]  

(1)

where \( F_0 \) is a constant value reached for large lateral displacements, \( w \), and \( \mu_1 \) is a stiffness parameter.
FIG. 2 - LATERAL RESISTANCE CHARACTERISTIC

\[ \left( \frac{F}{F_0} \right) = \tanh\left( \frac{H}{W} \right) \]

- \( \mu_1 = 100\ M^{-1} = 2.54\ IN^{-1} \)
- \( \mu_1 = 500\ M^{-1} = 12.7\ IN^{-1} \)
- \( \mu_1 = \infty \)

INCHES

0
0.5
1.0
1.5
2.0

METERS

0
0.01
0.02
0.03
0.04
0.05

FIG. 3 - LONGITUDINAL RESISTANCE CHARACTERISTIC

\[ \left( \frac{F}{F_0} \right) = \tanh\left( \frac{H_2}{U} \right) \]

- \( \mu_2 = 100\ M^{-1} = 2.54\ IN^{-1} \)
- \( \mu_2 = 200\ M^{-1} = 5.08\ IN^{-1} \)
- \( \mu_2 = 500\ M^{-1} = 12.7\ IN^{-1} \)
- \( \mu_2 = 1000\ M^{-1} = 25.4\ IN^{-1} \)
- \( \mu_2 = \infty \)

INCHES

0
0.2
0.4
0.6
0.8

METERS

0
0.005
0.010
0.015
0.020

U
For service tracks the value of \( u_1 \) is large (typically greater than 12.7 per inch or 500 per meter). Rigorous calculations performed earlier by Samavedam [2] have shown that the safe temperature is not sensitive for a wide range of \( u_1 \). This can be expected also from the fact that the buckling displacements are so large that the initial part of the resistance curve will have negligible effect. This implies that the resistance can be approximated as a constant, \( F_o \), so that the lateral resistance function is given by:

\[
F(\omega) = \mp F_o \quad (\pm \text{depending on the sign of } \omega)
\]

A typical range of \( F_o \) is 12 to 112 lb/in. The effect of the lateral resistance parameter \( F_o \) on the buckling response will be presented in detail. For a method of determining \( F_o \) analytically from field data, refer to [4].

**Longitudinal Resistance**

This is the resistance experienced by the rail-tie structure against movement in the longitudinal direction. When rail anchors are tight, ties also move with rails. The longitudinal resistance in this case is mostly offered by the ballast. In some situations, anchors may be loose, rails slip over the ties and thus experience reduced resistance.

The longitudinal resistance is also a nonlinear function of the longitudinal displacement. Mathematically it can be represented by a function of the type (Figure 3)

\[
f(u) = f_o \tanh (u_2 u)
\]  

(2)

The longitudinal displacements experienced by the track during buckling are small. Therefore, the coefficient \( u_2 \) is expected to have some effect on the safe temperature. Calculations on the effect of \( u_2 \) performed by Samavedam [2] show that within a typical range for \( u_2 \) of between 1.27 to 50.8 per inch (50 to 2000 per meter), the safe temperature can vary by about 10%. No further study will be done here on the effect of \( u_2 \); the resistance will be idealized as a constant (implying \( u_2 \) is a large quantity), and is given by:
\[ f(u) = f_0 \]

Typical range of values for \( f_0 \) are also between 12 to 112 lb/in. The effect of the longitudinal resistance \( f_0 \) on the buckling response will also be presented. For a method to determine \( f_0 \) analytically from field tests, refer to [4].

**Torsional Resistance**

The torsional resistance is offered by the fasteners. The resistance, \( \tau \), is a nonlinear function of the twist or the rotation of the rails, \( w' \), given by

\[
\tau = \tau_0 \tanh(\mu_3 w')
\]

(3)

where, \( \tau_0 \) is a constant, \( \mu_3 \) a stiffness parameter and \( w' \) is the slope of the deflection, \( w \), i.e. \( w' = dw/dx \).

Calculations carried out previously by Samavedam [2] show that for resistance values of \( \tau_0 \) of the order of a few hundred kgf/m/rad, the effect on the safe temperature increase is negligible. For wood ties with cut spikes, the influence is expected to be even smaller. Of course there are fasteners with improved torsional resistance characteristics and higher torsional stiffness values as indicated in [12]. Unfortunately the data presented in [12] is of limited use for stability investigations because of the very small rotations used in the experiment. Additionally, the contribution of resistance against rotation offered by the ballast to the tie was not included, which may be a significant part of a panel's total torsional rigidity. The benefits of using high torsional stiffness fasteners on improving buckling strength are intuitively evident. The quantification of these benefits will be evaluated in a future study.

2.2 TRACK IMPERFECTIONS

Lateral misalignments are very important in stability considerations, and can be of numerous forms. Although the analyses developed in [2] are applicable to any form of imperfection, for simplicity, only "sinusoidal" imperfec-
tions will be addressed here. A symmetric imperfection can be represented by

$$w_0 = \delta_0 \cos \left( \frac{\pi x}{2L_0} \right) \quad \text{for} \quad |x| \leq L_0$$

(4)

where $\delta_0$ is the amplitude, and $2L_0$ is the length over which the imperfection is spread (Figure 4).

The effect of varying the amplitude and the length of imperfection on the buckling response will be presented later.

2.3 CURVATURE

It is well known that curved track is more sensitive to thermal buckling than tangent track. Generally, CWR on curves is restricted to a minimum radius of 600 m (about 3°) for most European railroads; however, U.S. railroads do not limit use of CWR in curvatures. Parametric studies on the British CWR indicate that if the radius of curvature is greater than 1000 m (1.4° curve), then the track can be approximated as a tangent track for normal service conditions. Thus, the radius of curvature in the range below 1000 m is of practical interest and its effect within this range on thermal buckling will be presented in this study.

2.4 RAIL PROPERTIES

Rail properties (material and sectional) will also have an effect on the track buckling response. The material properties of interest in the present studies are

- Young's modulus, $E$
- Coefficient of thermal expansion, $\alpha$

Generally, the variation in the modulus is not significant. However, certain rail steels containing manganese have reduced coefficients of thermal expansion and therefore higher buckling strength (increased by about 10% over ordinary rail steel). No further discussion on effects of material properties will be included here.
The effect of the rail sectional properties on the buckling response is of practical interest as rails of different sizes are in use. The sectional properties of interest are

- Cross-sectional area
- Moment of inertia about the vertical axis.

Since the rails are made in standard sections, it is not required to vary the two parameters (area and moment of inertia) independently. Rather, the weight of the rail per yard can be conveniently considered as a variable. The effect of this parameter on the buckling response will be also presented.

2.5 FINITE LENGTH AND STIFFNESS

CWR tracks are usually considered as infinitely long and uniformly heated. However, test tracks used in the buckling experiments are generally finite in length with some end restraints, as in the European tests, or, they may be long but heated over a finite length, as in the recent tests in the U.S. [4]. In both situations, the end "boundary" conditions should be stated for analytical predictions and for correct understanding of the observed buckling response of the experimental track. The parameters of interest here are

- Heated length
- End stiffness, (or stiffness at the cold and hot junction).

Kerr's [5] treatment of the buckling of "short" tracks did not include the influence of end restraint. A rigorous analysis has been recently developed by Kish, Samavedam and Jeong [4]. On the basis of this analysis, the effect of the track length and the end stiffness on buckling response is expected to differ from [5], as will be shown later.

2.6 ANALYTICAL FORMULATION

A general mathematical formulation for lateral buckling of infinite tracks (tangent and curved) has been presented earlier by Samavedam [2]. For finite
tracks, a recent treatment is given by Kish, et al. [4]. These formulations not repeated here, provide the basis for the present work. The assumptions in the analyses relevant to the current study will be stated for the sake of clarity.

• The analysis is based on the "track-beam" type static theory of buckling where vehicle induced forces and track inertia effects are not included. Validation studies for the static theory showed good agreement [4]. Dynamic aspects are treated in [8].

• Various post buckling shapes are possible as sketched in Figure 5 (see page 10). The shape is determined by the initial imperfections to some extent. Shape I mode is the simplest mode from the analysis point of view. Tangent tracks with symmetric Shape I type imperfection, starts buckling in this shape and may undergo a mode change into Shape III, as their final buckled shape. This was evidenced in a recent buckling test [4]. Curved tracks of relatively short radii tend to buckle out in Shape I. Most of the parametric study done here is based on Shape I analysis, although where relevant, other shapes are considered also.

• As done in earlier analyses, the longitudinal resistance in the buckled zone is neglected. For higher mode shapes (e.g., Shape III), this assumption may lead to some differences in the results. Calculations carried out by Samavedam [2] have shown that the safe temperature for Shape I with longitudinal resistance neglected in the buckled zone is more or less the same as that for Shape III with the resistance included in the buckled zone. This may justify the use of approximate Shape I analysis for the determination of safe temperatures in some cases. It has been also shown that the difference in the computed safe temperature increase with and without longitudinal resistance in the buckled zone is negligible.

• Also as treated in earlier analyses, torsional resistance of fasteners is neglected; and lateral and longitudinal resistances are taken to be constant. For the influence of nonlinearity in lateral and longitudinal resistances, see [2].
3. SENSITIVITY STUDY

The important parameters identified in the previous section for the purpose of parametric studies are shown in Table 1. Sensitivity analyses of each of the parameters will now be presented. Although numerous permutations are possible due to several parameters involved in the analyses, for brevity, only a limited set of results are presented. These are believed to adequately highlight the practical significance of the parameters. Each of the parameters is varied one at a time, while the others are kept constant at some realistic values.

3.1 EFFECT OF LATERAL RESISTANCE

The resistance is idealized as a constant, \( F_o \) (see Figure 2). It is measured per unit length of the track, expressed in pound/inch or kg/meter. It must be noted that this is not a stiffness measurement, as the magnitude of the force is independent of the level of the lateral deflection.

Figure 6 shows the variations of the buckling and the safe temperature increases for the infinite track over a range of lateral resistance values (to obtain absolute temperatures, add the rail neutral temperature to these temperature increases). The values of other parameters which are kept constant in the study are shown in the figure.

Figure 6 also shows that the buckling temperature \( \Delta T_B \) increases rapidly with the increase in the lateral resistance. The safe temperature also increases though at a much lower rate. At sufficiently low values of lateral resistance, the buckling and the safe temperatures no longer exist in tracks with imperfections, i.e., the track will progressively incur larger and larger deflections without actually buckling in an explosive manner. This is denoted as "progressive buckling."

Typical results for a curved track are shown in Figure 7. It is again noted that the lateral resistance has a significant influence both on the safe and the buckling temperature.
<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>IDEALIZED FORM</th>
<th>RANGE OF PRACTICAL INTEREST</th>
<th>APPLICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 LATERAL RESISTANCE</td>
<td>CONSTANT, $f_o$</td>
<td>12 - 112 LB/IN (200 - 2000 KG/M)</td>
<td>TRACK MAINTENANCE AND DESIGN</td>
</tr>
<tr>
<td>2 LONGITUDINAL RESISTANCE</td>
<td>CONSTANT, $f_o$</td>
<td>12 - 112 LB/IN (200-2000 KG/M)</td>
<td>TRACK MAINTENANCE AND DESIGN</td>
</tr>
<tr>
<td>3 LATERAL IMPERFECTION</td>
<td>$\delta_o \cos \frac{\pi x}{2L_o}$</td>
<td>$\delta_o$ (AMPLITUDE) 0 - 2 IN (0 - 50 MM)</td>
<td>TRACK MAINTENANCE</td>
</tr>
<tr>
<td>4 CURVATURE</td>
<td>CONSTANT (RADIUS, R)</td>
<td>2° - 6° (1000 - 300 M)</td>
<td>CURVED TRACK</td>
</tr>
<tr>
<td>5 RAIL PROPERTIES</td>
<td>STANDARD SECTIONS BY WEIGHT</td>
<td>100 LB - 140 LB</td>
<td>TRACK DESIGN</td>
</tr>
<tr>
<td>6 END STIFFNESS FOR FINITE TRACKS</td>
<td>LINEARIZED STIFFNESS CONSTANT, $k$</td>
<td>$5.5 \times 10^5$ - $5.5 \times 10^6$ LB/IN x</td>
<td>TEST TRACKS</td>
</tr>
</tbody>
</table>

*FOR TWO RAILS.*
FIG. 6 - EFFECT OF LATERAL RESISTANCE, $F_o$ (TANGENT)

FIG. 7 - EFFECT OF LATERAL RESISTANCE, $F_o$ (5° CURVE)
Figure 8 shows the effect of missing ties on the buckling response for a tangent track without imperfections. Missing ties are modeled by zero lateral resistance in the region where ties are absent. It is seen that if the track is designed on the basis of its safe temperature, one or two missing ties do not substantially lower the buckling strength.

3.1.1 Factors Governing Lateral Resistance

There are several factors which affect the value of the lateral resistance. Among them are the level of consolidation, the ballast shoulder width and height, the spacing of ties, the tie material and design, ballast, and environmental conditions.

A freshly tamped track exhibits a reduced lateral resistance anywhere around 33.5 lb/in (600 kg/m) (this number is usually recommended for "weak" track). A consolidated track can have anywhere over 56 lb/in (1000 kg/m). Both numbers are estimated values based on European experience.

The ballast shoulder can contribute to the overall resistance by 10-20%. There is a maximum width of the shoulder beyond which the increase in the resistance is not appreciable. Thus a 12" shoulder is better than a 6" one, but the difference between 18" and 24" may not be substantial. A quantified effect of the ballast shoulder on the resistance for the U.S. mainline tracks is not available to date.

The tie type, design and spacing also have an effect on the resistance. British Rail claims that concrete ties have higher resistance than the wood ties because of the increased bottom friction due to weight and roughened bottom surfaces. On the other hand, some researchers conjecture that after ballast consolidates, ballast tends to "dig" into wood tie bottoms, thereby increasing resistance to a level equal to that of concrete ties.

The tie spacing can be used by the engineer as a parameter to vary the lateral resistance to some extent. The British Rail recommended practice for tracks (carrying a speed limit of 100 mph) is to have a concrete tie spacing
FIG. 8 - EFFECT OF MISSING TIES
of 27.6 inches (700 mm) for tangent tracks, and 23.6 inches (600 mm) for curved tracks [11]. Reducing the spacing increases the lateral resistance up to a point, beyond which the effect may not be significant. To date, the optimum tradeoffs between the spacing, ballast shoulder width and curvature have not been established.

3.2 EFFECT OF LONGITUDINAL RESISTANCE

As stated earlier, this is the resistance to the rail movement in the longitudinal direction. If the anchors are tight, the resistance is due to the ballast. As in the case of the lateral resistance, the longitudinal resistance is measured per unit length of the track (both rails are considered together).

The effect of longitudinal resistance on the safe and the buckling temperatures is seen in Figures 9 and 10 for a tangent and for a 5° curve, respectively. As seen in these figures, the safe temperature increases with the resistance value. Comparing the results in Figures 6 and 9, it may be concluded that the safe temperature is more sensitive to the lateral resistance than to the longitudinal resistance.

Figures 9 and 10 also indicate that the buckling temperature is insensitive to the longitudinal resistance. This is in contrast to the situation in Figure 6, in which it is seen that the lateral resistance has a significant effect on the buckling temperature.

The negligible effect of the longitudinal resistance on the buckling temperature as seen here can be attributed to certain simplifying assumptions made in the analysis, e.g., resistance is negligible in the buckling zone. Rigorous calculations can be performed without resorting to the simplifying assumptions. Such calculations would show some sensitivity of the buckling temperature to the longitudinal resistance, although this sensitivity is expected to be small for the infinite track, as the longitudinal movement is negligible before the onset of buckling.

The foregoing discussion does not imply that the longitudinal resistance
FIG. 9 - EFFECT OF LONGITUDINAL RESISTANCE, $f_o$ (TANGENT)

FIG. 10 - EFFECT OF LONGITUDINAL RESISTANCE, $f_o$ (5° CURVE)
plays a minor role in improving the overall stability of CWR tracks. As stated earlier it has an important effect on the safe temperature increase. Furthermore, the following benefits are obtained by increasing the longitudinal resistance:

- Due to nonuniform heating (sun and shade) there will be gradients of the thermal force along the track. The rails will develop a tendency to creep through fasteners, which when it occurs, will contribute to changes in the neutral temperature.

- High longitudinal resistance will also enable the track to resist traction and braking forces. This will reduce the possibility of "bunching of rails" in one direction due to repeated braking actions, hence, the changes in the rail neutral temperature are minimized.

- Changes of neutral temperature imply loss of neutral temperature at some spots and gain at other spots. Loss of neutral temperature is a direct reduction in buckling strength, i.e. both $\Delta T_B$ and $\Delta T_s$ in effect are reduced.

- It can be shown that the external energy needed to distort the track at a given temperature (below the buckling temperature) is reduced with decrease in longitudinal resistance. Hence the "degree of stability" increases with increase in the longitudinal resistance.

- The formation of sun kinks and small track distortions can occur due to inadequate anchoring, particularly in the presence of vehicle loads and high thermal forces. These will precipitate into larger and more critical size lateral track misalignments, which, in turn, can reduce the buckling temperature, $\Delta T_B$. (It must be noted that in Figures 10 and 11, the imperfections are assumed to be caused by other factors not related to inadequate anchoring, i.e. weakened longitudinal resistance).

3.2.1 Factors Governing Longitudinal Resistance

The resistance is derived mostly from the crib ballast and the tie bottom friction, and is transferred to the rails through the fasteners or rail
anchors. The longitudinal resistance is provided to tracks to withstand traction and braking forces and help maintain the rail neutral temperature as close as possible at the installation temperature, thereby increasing the buckling strength.

For wood tie tracks, rail anchors are used either on every alternate tie or on each tie. The resistance in the latter situation is almost twice the value for the former. Sometimes anchors are used only on one side of the ties. In this case, the resistance will not be the same in both directions.

3.3 EFFECT OF IMPERFECTIONS

Track imperfections, or alignment deviations, can be of numerous forms. For simplicity, only sinusoidal misalignments will be considered which are defined by an amplitude or offset, δ₀, and by a wavelength, 2L₀.

The effect of misalignment amplitude on the safe and the buckling temperatures is shown in Figure 11. One pair of curves represents the tangent track and the other the curved track. The values of the resistance parameters used in the numerical study are those measured for the buckling test tracks at The Plains, VA (see Kish, et al. [4]).

As seen in Figure 11, with increase in the amplitude, the buckling temperature quickly reduces. The safe temperature also decreases, though not as severely. At sufficiently large amplitudes, the buckling and the safe temperatures coalesce. For this and larger amplitudes, the track buckles out progressively, and no explosive buckling occurs.

The effect of "wavelength" of the misalignment on the buckling and the safe temperatures are shown in Figure 12. From this it may be seen that the buckling temperatures decrease rapidly as the misalignment length reduces. Hence, it is concluded that sharp localized imperfections are more dangerous than those spread over longer lengths.
$F_o = 1490 \text{ kg/m} = 83.3 \text{ lb/in}$

$f_o = 1560 \text{ kg/m} = 87.2 \text{ lb/in}$

$2L_o = 11 \text{ m} = 36.1 \text{ ft}$

FIG. 11 - EFFECT OF MISALIGNMENT AMPLITUDE
TANGENT

$F_0 = 1490 \text{ kg/m} = 83.3 \text{ lb/in}$

$f_o = 1560 \text{ kg/m} = 87.2 \text{ lb/in}$

$\delta_o = 0.025 \text{ m} = 0.98 \text{ in}$

FIG. 12 - EFFECT OF MISALIGNMENT LENGTH (TANGENT)
3.3.1 Factors Influencing Lateral Misalignments

One of the many causes for incurring lateral misalignment is due to improper welds. These tend to result in sharp localized imperfections at the welded junctions.

On hot days, the lateral forces exerted by the moving vehicles on track can cause some shift. After the wheel passage, the track may not fully return to its original position due to

- inelastic behavior of ballast in conjunction with a high lateral load (e.g., track displaced laterally 2 inches will recover only about 1/2 inch, leaving a permanent set of 1.5 inches). Since the limit of elastic behavior can be as low as .040 to .120 inches (1 to 3 mm), any deflection longer than these could result in a permanent set.

- compressive forces in the rails due to solar heating may be adequate to constrain the track in the new equilibrium position.

The initial imperfections can grow as the temperature rises, ending up in the so-called sun kinks. The exact mechanism of sun kink formation and the effect of vehicle induced loads are currently under investigation.

3.4 EFFECT OF CURVATURE

Although practical curves have segments of varying radii, for the purpose of analysis, it is sufficient to consider the segment with minimum radius. Buckling half wave length is generally no more than 30-50 ft (10-15 meters).

Conventional U.S. railroad practice is to express curves in degrees, $\theta$, as opposed to European practice of radii definitions. The radius $R$ is given by the formula according to Hay [9] as:

$$ R = \frac{100}{\theta} \quad (5) $$

where $R$ is given in feet and $\theta$ in radians ($2\pi$ radians = 360°).
Some results for curved tracks are already presented in Figures 7, 10 and 11. The results of the curved track may be compared with those of the tangent track (Figure 11) to understand the relative sensitivity of the two tracks to imperfections. For given imperfection and resistance parameters, the curved track has lower safe and buckling temperatures, as one would expect. Whereas the safe temperature difference between the curve and the tangent is small (20% for small imperfections) the buckling temperature difference is significantly large. Therefore, curved tracks are more vulnerable to buckling than the tangent tracks.

Figure 13 gives the results for the safe and the buckling temperature increases for a range of track radii. The results for the tangent track are included as the infinite radius case. It is seen from the figure that the temperatures decrease with increasing curvature, i.e., decreasing radius. The buckling temperatures are particularly sensitive to the curvature.

It is also noted in Figure 13 that for large curvature (short radii) tracks, the buckling and the safe temperatures coalesce, or may not be clearly identified in the response diagram. This condition implies that such curves simply move outwards progressively with increase in temperature, and buckling in the explosive sense will not take place. Thus, large curvature tracks ("tight curves") are susceptible to progressive lateral movements due to thermal forces in the rails.

3.4.1 Thermal Problems with Curves

Curves are sensitive to thermal forces; they "breathe" in and out with temperature fluctuations. Problems can also occur in reverse curves and spirals due to the uneven force buildup. Additionally, at temperatures below the stress-free temperature (especially in winter), the tensile forces will tend to "tighten" or pull the curve inward.

In some cases, the curving forces generated by traffic are adequate to shift the curve outwards, particularly when the rails are in compression. The prevention of track lateral motion (sun-kinks) through adequate lateral resistance and controlled thermal and curving forces is an important research prob-
\[ F_o = 972 \text{ KG/M} = 54.3 \text{ LB/IN} \]
\[ f_o = 1240 \text{ KG/M} = 69.3 \text{ LB/IN} \]
\[ \delta_o = 0.041 \text{ M} = 1.61 \text{ IN} \]
\[ 2L_o = 11 \text{ M} = 36.1 \text{ FT} \]

**FIG. 13 - EFFECT OF CURVATURE**
lem for U.S. track. Analytical and experimental efforts by the French National Railways (SNCF) [10] attempted to establish limiting values of lateral forces for the prevention of sun-kinks, however, applicability to practical design considerations became difficult.

3.5 EFFECT OF RAIL SECTION

The buckling and the safe temperatures for different rail sections are shown in Figure 14 for typical set of resistance values and imperfection. The properties of different rail sections considered are shown in Table 2. It is seen from Figure 14 that the smaller the weight of the rail, the stronger the track will be from the thermal stability point of view. It must be remarked that the lateral resistance also changes somewhat as the rail weight is varied due to the tie bottom friction being influenced by the tie self weight and the portion of superimposed rail weight. It should also be noted from Figure 14 that although a substantial difference in buckling strength is evident in comparing 100 lb rail to 70 lb rail there is only a small difference apparent in contrasting 140 lb and 115 lb, which is the typical rail weight range for CWR in the U.S.

3.6 EFFECT OF TRACK LENGTH AND END STIFFNESS

Many buckling tests in the past were performed on tracks heated over finite lengths. As stated earlier, the buckling response of tracks is influenced by the heated length and the end stiffness.

The main problem associated with any finite length heated test track is the development of prebuckling longitudinal displacement. At a given temperature, the prebuckling end displacement is determined by the "end stiffness" and the longitudinal resistance. This is in contrast with the situation of the infinite track which has no prebuckling movements regardless of the value of its longitudinal resistance.

Figure 15 shows a quantitative assessment of the end displacement of a 656 ft (200 meter) long track for different values of end stiffness, k, by using the finite track analysis described in [4].
TANGENT

\( F_p = 600 \text{ KG/M} = 33.5 \text{ LB/IN} \)

\( F_p = 1000 \text{ KG/M} = 55.9 \text{ LB/IN} \)

\( \delta_0 = 0.025 \text{ M.} = 0.98 \text{ IN} \)

\( L_0 = 5.0 \text{ M.} = 16.4 \text{ FT} \)

FIG. 14 - EFFECT OF RAIL SECTION
TABLE 2 - RAIL PROPERTIES

YOUNG'S MODULUS, $E = 30 \times 10^6$ PSI

COEFFICIENT OF THERMAL EXPANSION, $\alpha = 6.4 \times 10^{-6}/^\circ F$

$(1.16 \times 10^{-5}/^\circ C)$

<table>
<thead>
<tr>
<th>RAIL SIZE AND SECTION</th>
<th>MOMENT OF INERTIA WITH RESPECT TO VERTICAL AXIS*</th>
<th>CROSS SECTIONAL AREA*</th>
</tr>
</thead>
<tbody>
<tr>
<td>70 LB ASCE $^{(1)}$</td>
<td>$9.72 \text{ IN}^4$ ($404 \times 10^{-8} \text{ M}^4$)</td>
<td>$13.62 \text{ IN}^2$ ($87.88 \times 10^{-4} \text{ M}^2$)</td>
</tr>
<tr>
<td>115 LB AREA $^{(2)}$</td>
<td>$21.6 \text{ IN}^4$ ($900 \times 10^{-8} \text{ M}^4$)</td>
<td>$22.5 \text{ IN}^2$ ($145.6 \times 10^{-4} \text{ M}^2$)</td>
</tr>
<tr>
<td>132 LB AREA $^{(2)}$</td>
<td>$29.2 \text{ IN}^4$ ($1214 \times 10^{-8} \text{ M}^4$)</td>
<td>$25.8 \text{ IN}^2$ ($167.1 \times 10^{-4} \text{ M}^2$)</td>
</tr>
<tr>
<td>140 LB AREA $^{(2)}$</td>
<td>$29.6 \text{ IN}^4$ ($1232 \times 10^{-8} \text{ M}^4$)</td>
<td>$27.6 \text{ IN}^2$ ($178.96 \times 10^{-4} \text{ M}^2$)</td>
</tr>
</tbody>
</table>

*PROPERTIES PER TWO RAILS.


FIG. 15 - DEPENDENCE OF END MOVEMENTS ON STIFFNESS
A problem that arises in the design of buckling tests is the determination of minimum length required to simulate the infinite track. If the criterion is based on the safe temperature increase, then it can be shown that the minimum length required is given by:

\[ 2L = 2(L + l_b + l_s) \]  

(6)

where \( 2L \) = buckling length of the infinite track
\( 2l_b \) = breathing length
\( 2l_s \) = the length of the track experiencing prebuckling displacements due to finite stiffness, \( k \), at the ends

and that \( l_s \) and \( l_b \) are given by:

\[ l_s = \left( \frac{AE}{k} \right) \left[ \sqrt{1 + \frac{2k\alpha T}{f_o}} - 1 \right] \]

\[ l_b = \frac{AEa - \bar{F}}{f_o} \]

(7)

It is clear that the effect of prebuckling longitudinal displacements can be minimized by

- Increasing \( k \), the end stiffness, by providing large end restraint, such as via massive concrete blocks as done in the British Rail tests
- Increasing the longitudinal resistance, \( f_o \), by anchoring every tie and increasing the amount of ballast
- Providing a large imperfection and a reduced lateral resistance to decrease the buckling temperature \( T \).

Table 3 presents the values for lengths \( 2L \), \( 2l_b \), and \( 2l_s \) for two values of the end stiffness parameter.

Figure 16 presents the variation of the safe temperature increase with track length for Shape II mode of buckling. The case of large end stiffness
### TABLE 3 - TEST SECTION LENGTH REQUIREMENT

<table>
<thead>
<tr>
<th>$k$ (END STIFFNESS)</th>
<th>LATERAL RESISTANCE $F_0$ (LB/IN)</th>
<th>$\Delta T_a$ (SAFE TEMP. FOR INFIN. TRACK $^oF$)</th>
<th>$\bar{P}$ (FORCE INBUCKLED ZONE TONS)</th>
<th>$L$ (BUCKLING LENGTH FT.)</th>
<th>$t_b$ (BREATHING LENGTH FT.)</th>
<th>$t_a$ (SPRING INFLUENCE)</th>
<th>TOTAL TRACK LENGTH $2t_a - 2(t_b + t_a)$ FT.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.6 \times 10^3$ (1000)</td>
<td>55.9 (46.4)</td>
<td>83.5 (122410)</td>
<td>134.9 (6.5)</td>
<td>221.1 (67.4)</td>
<td>281.2 (85.7)</td>
<td>1047.2 (319.2)</td>
<td></td>
</tr>
<tr>
<td>$5.6 \times 10^3$ (1500)</td>
<td>83.8 (54.0)</td>
<td>97.2 (143660)</td>
<td>158.4 (6.0)</td>
<td>253.5 (77.2)</td>
<td>308.4 (94.0)</td>
<td>1162.7 (354.4)</td>
<td></td>
</tr>
<tr>
<td>$5.6 \times 10^6$ (1000)</td>
<td>55.9 (46.4)</td>
<td>83.5 (122410)</td>
<td>134.9 (6.5)</td>
<td>221.1 (67.4)</td>
<td>108.9 (33.2)</td>
<td>702.8 (214.2)</td>
<td></td>
</tr>
<tr>
<td>$5.6 \times 10^6$ (1500)</td>
<td>83.8 (54.0)</td>
<td>97.2 (143660)</td>
<td>158.4 (6.0)</td>
<td>253.5 (77.2)</td>
<td>118.4 (36.1)</td>
<td>782.8 (238.6)</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** ABOVE VALUES ARE FOR 132 LB. RAIL AND A LONGITUDINAL RESISTANCE, $f_o$, OF 55.9 LB/IN.
**SHAPE II**

TANGENT 132 LB RAIL

\( F_e = 900 \text{ KG}/\text{M} = 50.3 \text{ LB}/\text{IN} \)

\( f_e = 1000 \text{ KG}/\text{M} = 55.9 \text{ LB}/\text{IN} \)

**Fig. 16** - EFFECT OF TRACK LENGTH AND END STIFFNESS ON SAFE TEMPERATURE ON MODE SHAPE II
(K = 0), i.e., fixed end conditions, has been previously studied by Kerr [5]. A practical value for k is on the order of 5.6 \times 10^5 \text{ lb/in} (10^7 \text{ kg/m}). For short test tracks (say 328 feet or 100 meters), the error contributed by low end stiffness can be significant. Hence, the test track lengths recommended by Kerr tend to be unconservative.

Additional results for tangent and curved tracks are shown in Figure 17 for Shape I. It appears that the tangent track is slightly more sensitive to the end stiffness effects than the curved track.

The end stiffness can also have significant effect on the buckling temperature, particularly for short tracks. Figures 18 and 19 give the buckling response for a track 328 feet (100 meter) long with two different sizes of lateral misalignments. It is seen that the buckling temperatures increase rapidly with decreasing end stiffness. Also, at the low stiffness value of 5.6 \times 10^6 \text{ lb/in} (10^6 \text{ kg/m}), the buckling temperature is very large as seen in Figure 19, indicating that at low end stiffness, the track simply elongates in the longitudinal direction, without building up sufficient compressive forces. It can be concluded that, in general, buckling tests on short tracks can misrepresent reality and lead to overestimates of CWR buckling strength, unless the results are sensibly interpreted in the light of present theoretical considerations.
FIG. 17 - EFFECT OF TRACK LENGTH AND END STIFFNESS ON SAFE TEMPERATURE ON MODE SHAPE I
SHAPE I
132 LB RAIL
TRACK LENGTH = 100 M = 328 FT
TANGENT
\( f_o = 600 \text{ KG/M} = 33.5 \text{ LB/IN} \)
\( \delta_o = 1000 \text{ KG/M} = 55.9 \text{ LB/IN} \)
\( \delta_o = 0.025 \text{ M.} = 0.98 \text{ IN} \)
\( L_o = 5.0 \text{ M.} = 16.4 \text{ FT} \)

FIG. 18 - EFFECT OF END STIFFNESS ON BUCKLING RESPONSE
SHAPE I
132 LB RAIL
TRACK LENGTH = 100 M = 328 FT
TANGENT
\( F_0 = 600 \text{ KG/M} = 33.5 \text{ LB/IN} \)
\( f_0 = 1000 \text{ KG/M} = 55.9 \text{ LB/IN} \)
\( \delta_0 = 0.040 \text{ M.} = 1.574 \text{ IN} \)
\( L_0 = 5.0 \text{ M.} = 16.4 \text{ FT} \)

FIG. 19 - EFFECT OF END STIFFNESS ON BUCKLING RESPONSE
4. DESIGN ASPECTS

From the general study presented in the previous sections, it may be seen that track incorporating continuous welded rail can be designed either on the basis of the buckling temperature, or on the safe temperature increase values. As shown in the parametric studies, the buckling temperature is very sensitive to changes in lateral resistance and to track imperfections, hence, tracks constructed according to designs based on the buckling temperature may be difficult to control.

The alternative criterion based on the safe temperature increase is simpler to use. The safe temperature is largely dependent on the lateral resistance and not that severely on other parameters. However, the safe temperature is still a conceptual parameter, not easily verifiable from tests, and needs to be established for dynamic vehicle loads.

If we assume that there is a factor of safety $f_s$ on the "static" safe temperature increase, the design criterion from the thermal buckling point of view can be written as

$$ (T_M - T_N) \leq f_s \Delta T_s $$

Here $T_M = \text{expected maximum temperature in the yearly cycle}$

$T_N = \text{neutral temperature}$

At the neutral temperature, the rails are expected to have zero resultant longitudinal force in the cross-section. Unfortunately, this may not be the track installation temperature. It is known that the following factors can influence the neutral temperature [6]

- Inefficient Installation/Destressing
- Operating Conditions
  - (i) Braking Effect
  - (ii) Tonnage ("Rolling-out")
- Track Configuration/Type
- Track Settlement and Heave
Currently there is no information available on the variation of the neutral temperature for the CWR in the U.S. quantifying the effect of the above factors. If we assume that the variation of the neutral temperature from the initial installation temperature can be accounted by an empirical factor $f_N$ (which depends on the above factors), then the anticipated minimum neutral temperature in the service life can be written as

$$T_N = f_N T_1$$

where $T_1$ is the installation temperature.

Hence, the design criterion can be written as

$$T_M - f_N T_1 \leq f_s \Delta T_s$$  \hspace{1cm} (9)

To increase the safety of CWR from the buckling point of view, the railroads can adopt the following measures:

- Increase the installation or destressing temperature $T_1$. For most railroads, the current practice is to aim for 80°F for the installation temperature. Further increase in the temperature may not be desirable, as it could lead to pull-aparts in winter due to large tensile stresses developed. However, it may be possible to follow this technique by "double destressing" the track in a year, once before summer and a second time before winter. Thus, the maximum compressive force in summer and the maximum tensile force in winter can be reduced by variable neutral temperature. Such a procedure is followed in Siberia by the USSR Railway. Even if this practice is adopted, it must be assumed that the neutral temperature change during each "destressing period" is minimal.

- Increase the safe temperature, $\Delta T_s$. From the parametric study discussed in earlier sections, this can be effected by:

  (i) increasing lateral and longitudinal resistances of the track
  (ii) minimizing imperfections
(iii) limiting track curvatures to less than 3° when possible (this design practice is currently utilized by several European railroads), however, this may not be realistic for U.S. track.
(iv) decreasing the rail size (if buckling safety outweighs other safety issues).

- Operational constraints such as slow orders or nighttime operation.
- "Pretensioning" the rails during installation.

Allowable safe temperature increase values can be easily plotted in the form of contours as shown in Figures 20 and 21 for 132 lb rail and various curvatures if approximate values of the longitudinal and lateral resistances are available. As indicated in the figures, "average" track falling in the resistance ranges of 45 lb/in to 67 lb/in exhibits an allowable safe temperature increase of 67°-85°F for a 5° curve. This implies that in a region where the maximum anticipated rail temperature is 140°F, for example, rail laying temperature cannot be lower than 73°F provided no reduction in track resistance and neutral temperature occurs.

The track supervisor must assure adequate resistances for his track, which, in general, will depend on the type of ties, tie spacing, type of fasteners, anchors, ballast type and condition, and consolidation level. Unfortunately, characterization of track resistance for buckling strength assessments is rather scarce for the U.S. railroad tracks, and although some information is available from European measurements, in general, it is not applicable to the U.S. tracks. Recently obtained U.S. data is summarized in Table 4.

The safe temperature increase values referred to in Figures 20 and 21 are allowable temperature increases above the rail neutral temperature. As discussed earlier, the neutral temperature is known to change due to a combination of factors such as tonnage, operating conditions, inadequate destressing, "breathing," and seasonal variations. These could induce a reduction in the neutral temperature by 30°F or more from its initial installation temperature. A 30°F reduction in neutral temperature could be extremely critical and easily lead to buckling since now the "apparent" laying temperature is 43°F for a
FIG. 20 - SAFE TEMPERATURE INCREASE FOR TANGENT AND 3° CURVED TRACK
FIG. 21 - SAFE TEMPERATURE INCREASE FOR 5° AND 7° CURVED TRACK
### TABLE 4 - TRACK RESISTANCE SUMMARY FROM U.S. TESTS

**READVILLE, MA**

<table>
<thead>
<tr>
<th>Item</th>
<th>Details</th>
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</thead>
<tbody>
<tr>
<td>Rail</td>
<td>131 Re Jointed</td>
</tr>
<tr>
<td>Ties</td>
<td>Wood, 20 in. Spacing, Fair To Poor Condition</td>
</tr>
<tr>
<td>Rail Anchor</td>
<td>Every Other Tie</td>
</tr>
<tr>
<td>Ballast</td>
<td>Traprock, 8-12 in. Shoulder Width</td>
</tr>
<tr>
<td>Use</td>
<td>Passenger Service</td>
</tr>
</tbody>
</table>

Lateral Resistance = 74 lb/in
Longitudinal Resistance = 61 lb/in (109 lb/in*)

**THE PLAINS, VA**

<table>
<thead>
<tr>
<th>Item</th>
<th>Details</th>
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<tbody>
<tr>
<td>Rail</td>
<td>132 Re CWR</td>
</tr>
<tr>
<td>Ties</td>
<td>Wood, 20 in. Spacing, Good Condition</td>
</tr>
<tr>
<td>Rail Anchor</td>
<td>Every Other Tie</td>
</tr>
<tr>
<td>Ballast</td>
<td>Granite, 10-14 in. S.W. (Tangent); 12-16 in. S.W. (Curved)</td>
</tr>
<tr>
<td>Use</td>
<td>Freight, Approx. 1.2 Mgt. Per Year</td>
</tr>
</tbody>
</table>

Tangent

Lateral Resistance = 54.3 lb/in
Longitudinal Resistance = 69.3 lb/in

Curved

Lateral Resistance = 83.3 lb/in
Longitudinal Resistance = 87.2 lb/in

*Every Tie Box Anchored*
rail that was laid at 73°F. Data and information on the subject of rail neutral temperature variation for the U.S. railroads are lacking. Some data has been collected by the British Rail, which may not be applicable for the U.S. tracks. It is hoped, however, that future research projects will cover this most important topic.

In addition to the critical issues of track resistance and rail neutral temperatures variation, dynamic effects on the safe temperature increase must also be determined. The importance of dynamic influences is evident from buckling occurrences under moving trains, and from the European test data [7]. It is necessary, therefore, to account for this influence on the safe temperature increase for improved design of CWR track. Once this influence is known, it can easily be "lumped" into the $f_s$ factor appearing in equation (8).

Until more data and information are available on track resistance characterization, rail neutral temperature variation and dynamic influences, consideration must be given to interim preventive measures such as improved and perhaps more frequent destressing, a sufficient margin of safety in CWR installation, ensuring adequate track resistance, and imposing speed and operational restrictions during extremely hot days especially for tracks with curvature greater than 5°.
5. CONCLUSIONS

- The important parameters which govern the buckling response of the tangent CWR track are the lateral resistance, the longitudinal resistance, and the lateral misalignment. The safe temperature increase is largely influenced by the first two parameters, whereas the buckling temperature is largely controlled by the first and the third parameters.

- The degree of curvature has a significant influence on the buckling strength of curved tracks. Both buckling and safe temperatures decrease with an increase in curvature; the former is more sensitive to the curvature than the latter.

- Buckling of short test tracks (or long tracks heated over finite lengths) are significantly influenced by the "end stiffness" and the heated length of the track.

- The relative influence of various parameters on the safe and the buckling temperature increases are shown in Table 5.

- In the design and maintenance of CWR track, rail neutral temperature is an important parameter since its variation can be significant. Buckling can occur if the neutral temperature drops to an unduly low value (say to 40°F from the installation value of 80°F). This problem can be more severe in curved tracks.

- The safe temperature increase should provide an adequate criterion for the design of CWR track if proper adjustments are included for rail neutral temperature variation and for dynamic influences. These adjustments are currently under investigation.
<table>
<thead>
<tr>
<th>PARAMETER</th>
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<th>EFFECT ON $\Delta T_S$</th>
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<tr>
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<td>HIGH</td>
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<td>FINITE TRACK</td>
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<td></td>
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REFERENCES


